

College of Engineering, Pune
End-Semester – May 2011
S. Y. B. Tech. Electrical
(MA 222 Mathematics IV: Signals and Systems)

Day & Date: –Saturday, 5 May 2012

Maximum Marks: – 50

Time: – 9:00 am to 12:00 noon

Duration: – 03.00 Hrs

Instructions:-

1. Read all the questions carefully.
2. Use of programmable calculator is strictly not allowed.
3. Assume suitable data where ever necessary.

Q. 1 Explain the bilinear transformation method for conversion of analog filter to digital filter. Convert the analog filter with system function, (5)

$$H_a(s) = \frac{s+0.1}{(s+0.1)^2+16}$$

into digital IIR filter by means of bilinear transformation.

The digital filter is to have a resonant frequency $\omega_r = \frac{\pi}{2}$.

Q. 2 A) If the Nyquist rate of a signal $x(t)$ is Ω_0 , then determine the Nyquist rate for the following signals: (3)

(i) $x(t) + x(t-1)$

(ii) $x(t)\cos\Omega_0(t)$

(iii) $\frac{dx(t)}{dt}$

B) Consider the periodic CT signal $x(t)$ with fundamental period $2T_0$. $x(t)$ is defined over one period as, (2)

$$x(t) = At^2, \quad 0 \leq t < T_0$$
$$0, \quad T_0 \leq t < 2T_0$$

Let a_k be the Fourier series coefficients of $x(t)$. Suppose $a_0 = 1/6$ and $\sum_{-\infty}^{\infty} |a_k|^2 = \frac{1}{T_0}$. Determine the numerical values of the parameters A and T_0 .

Q. 3 Consider the stable CT LTI system described by:

$$\frac{d^2 y(t)}{dt^2} + \frac{5}{2} \frac{dy(t)}{dt} - \frac{3}{2} y(t) = x(t) \quad (5)$$

(a) Find the frequency response of the system.

(b) Find the output when the inputs are:

(i) $x_i(t) = \cos(3\pi t)$

(ii) $x_{ii}(t) = \sin\left(\frac{3\pi}{4}t\right) + \cos\left(\pi t + \frac{\pi}{3}\right)$

(c) Let $x_c(t)$ be an input signal with fundamental period T and Fourier series coefficients a_k . Write the Fourier series coefficients b_k of the corresponding output signal $y_c(t)$ in terms of a_k and the frequency response. Assume that the fundamental period of $y_c(t)$ is also T . Compare this method to finding the impulse response $h(t)$ and convolving $h(t)$ with the inputs to find the outputs.

Q. 4 An LTIDT system is specified by the equation,

$$y[n] + 2y[n-1] + y[n-2] = x[n] + x[n-1] \quad (5)$$

The input is $x[n] = \left(\frac{1}{2}\right)^n u[n]$. Determine

(i) Natural response for the initial conditions $y(-1) = y(-2) = 1$

(ii) Forced response

(iii) Total Response

Q. 5 A) Given the z-transform:

$$X(z) = \frac{4z^2 + \frac{1}{2}z}{z^2 - \frac{1}{6}z - \frac{1}{6}} \quad (3)$$

For each of the following ROC's, determine the corresponding signal $x[n]$

(i) $|z| < \frac{1}{3}$

(ii) $\frac{1}{3} < |z| < \frac{1}{2}$

(iii) $|z| > \frac{1}{2}$

B) Write the relation between DFT and DTFT, also write the relation between DFT and 'Z' transform. (2)