College of Engineering, Pune

(An Autonomous Institute of Government of Maharashtra, Pune-411005)

End Semester Exam

(MA 201) Engineering Mathematics III

Autumn Semester 2011-12

Programme: S.Y.B.Tech. Branches: All

Academic Year: 2011-12 Date: 21/11/11

Duration: 3 Hrs Max. Marks: 50

Instructions:

1. All questions are compulsory. 2. All symbols have their usual meanings.
3. Figures to the right indicates full marks. 4. Assume suitable data, if necessary.

Q.I Attempt the following: (10)

(a) Solve: $x\ dy = (x^5 + x^3y^2 + y)\ dx$.

(b) Find the orthogonal trajectories of $\dfrac{x^2}{2} + \dfrac{y^2}{2-b} = 1$.

(c) Show that e^x, e^{-x}, e^{2x} are linearly independent solutions of the differential equation $y''' - 2y'' - y' + 2y = 0$ and hence write the general equation.

(d) Solve: $p - q = log(x + y)$.

(e) Show that the vector function $\overline{F} = \dfrac{1}{r}\ [r^2\ \overline{a} + (\overline{a}\ .\ \overline{r})\ \overline{r}]$ is irrotational, where $\overline{a}$ is a constant vector.

Q.II Attempt the following:

(a) (*i*) Solve: $(y^4 + 2y)\ dx + (xy^3 + 2y^4 - 4x)\ dy = 0$. (3)

(*ii*) Reduce to Linear Differential Equation and solve: $xy(1 + xy^2)\dfrac{dy}{dx} = 1$. (3)

(b) Form a differential equation for RC circuit with e.m.f. $E(t)$, where R and C are constants and show that $I(t) = e^{\frac{-t}{RC}} \left[\int \dfrac{e^{\frac{t}{RC}}}{R} \dfrac{dE}{dt} dt + C_1 \right]$, C_1 is the constant of integration. Hence determine the current at time $t > 0$ in a RC circuit with $R = 10$ ohms, $C = 2 \times 10^{-4}$ farads and $E = 100$ volts given that initial current is 10 amp. (4)

OR

[P.T.O.]

(b) A beam of length $2l$ has its ends clamped at the two ends and its weight is W Kg per meter length. The deflection y at a distance x from one end is governed by the equation $\frac{d^2y}{dx^2} = \frac{W}{6\,El}(2\,l^2 - 6\,lx + 3x^2)$. Find the deflection curve using the conditions $y = 0$ and $\frac{dy}{dx} = 0$ at $x = 0$. Also find the deflection at the center of the beam. (4)

Q.III Attempt the following:

(a) (*i*) Form a Partial Differential Equation by eliminating an arbitrary function ϕ from $z = x^2\,\phi(x - y)$. Is it Lagrange's Partial Differential Equation ? (2)
(*ii*) Solve: $(z^2 - 2\,yz - y^2)p + (xy + zx)q = xy - zx$. (3)

OR

(a) A bar of length 10 cm has its ends A and B kept at 20^0 and 40^0 respectively until steady-state conditions prevail. The temperature at A is then suddenly raised to 50^0 and at the same time at B is lowered to 10^0. Find the subsequent temperature distribution. (5)

(b) Solve: $x^2\,y'' + 4\,xy' + 2y = x + sin\,x$. (5)

Q.IV Attempt the following:

(a) Show that the solution of the radio equation $V_{xx} = LC\,V_{tt}$ is
$V(x,t) = \sum_{n=1}^{\infty}\left(A_n\,cos\frac{n\pi}{l\sqrt{LC}}\,t + B_n\,sin\frac{n\pi}{l\sqrt{LC}}\,t\right)\,sin\frac{n\pi x}{l}$ assuming that $V = 0$ at the ends, $x = 0$ and $x = l$ for all t. Hence solve it when initial voltage V is $V_0\,sin\frac{\pi x}{l}$, $V_t(x,0) = 0$. (5)

OR

(a) Find the steady-state temperature distribution in a thin rectangular metal plate $0 < x < a$, $0 < y < b$ with its two faces insulated (so that the flow is two dimensional) with the following boundary conditions prescribed on the four edges:
$u(0,y) = u(a,y) = u(x,0) = 0, u(x,b) = f(x) = 100$. (5)

(b) (*i*) Prove that Div $(\overline{u} \times \overline{v}) = \overline{v}$. Curl $\overline{u} - \overline{u}$. Curl $\overline{v}$ and hence calculate Div $(\overline{u} \times \overline{v})$ where $\overline{u} = y\,\overline{i} + z\,\overline{j} + x\,\overline{k}$, $\overline{v} = yz\,\overline{i} + zx\,\overline{j} + xy\,\overline{k}$. (3)
(*ii*) Find the directional derivative of $\phi = xy^2 + yz^3$ at the point $(2,-1,1)$ in the direction of the normal to the surface $x\,ln\,z - y^2 = -4$ at $(-1,2,1)$. (2)

Q.V Attempt the following:

(a) (*i*) Using line integral, compute the work done by the force $\overline{F} = (2y+3)\,\overline{i} + xz\,\overline{j} + (yz - x)\,\overline{k}$ when it moves a particle from the point $(0,0,0)$ to the point $(2,1,1)$

along the curve $x = 2t^2,\ y = t,\ z = t^3$. (3)

(ii) If $\phi(x, y),\ \psi(x, y),\ \phi_y$ and ψ_x be continuous in a region E of the xy-plane bounded by a closed curve C, then prove that

$$\oint_C (\phi\ dx + \psi\ dy) = \int\int_E \left(\frac{\partial\psi}{\partial x} - \frac{\partial\phi}{\partial y}\right) dx\ dy. \quad (2)$$

(b) Use Stoke's Theorem to evaluate $\oint_C \overline{F}.\ \overline{dr}$, where $\overline{F} = y^2\ \overline{i} + xy\ \overline{j} + xz\ \overline{k}$ and C is the boundary curve of the hemisphere $x^2 + y^2 + z^2 = 9$ and $z > 0$ oriented in the positive direction. (5)

OR

(b) State Gauss Divergence Theorem and use it to evaluate $\int\int \overline{F}.\ \hat{n}\ ds$, where $\overline{F} = 2x^2y\ \overline{i} - y^2\ \overline{j} + 4\ xz^2\ \overline{k}$ taken over the region in the first octant bounded by $y^2 + z^2 = 9$ and $x = 2$. (5)

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