

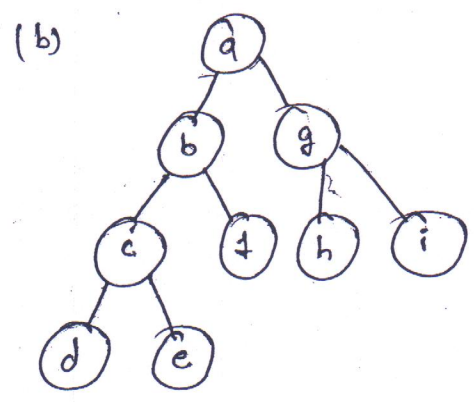
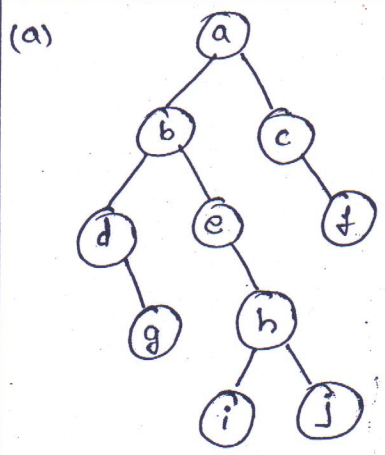
COLLEGE OF ENGINEERING, PUNE
 (An Autonomous Institute Of Government Of Maharashtra)
 End-Sem Examination
 (CT 201) DISCRETE STRUCTURE AND GRAPH THEORY

Programme: S.Y.B.Tech
 Year: 2011-12
 Duration: 3 hrs
 Instructions:

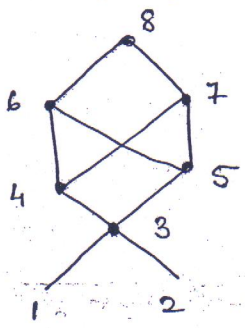
Semester: Autumn
 Max.Marks: 50

1. Draw the diagrams whenever required.
2. Explain with suitable example, whenever required.

SR. NO.	QUESTIONS with SOLUTIONS	MARKS
Qu1.		
A.	A connected planar graph has 9 vertices having degrees 2, 2,2,3,3,3,4,4 and 5. How many edges and faces are there?	2
ANS	$2e=2+2+2+3+3+3+4+4+5$ $e=14$ $v-e+r=2$ $9-14+r=2$ $r=7$	
B	A telephone network is established among 100 people. Information received by the first person is passed along to the 99 others as follows; the first person calls exactly 3 people, and each of these people calls 3 others, and so on until there are no others to call. If each call takes 5 minutes, how long does it take for a message to be relayed from the first person to receive the message to everyone else? How many people make no calls?	5
ANS	<p>The given situation can be represented by a tree</p> <p>Total no. of nodes- 100</p> <p>since each person exactly 3 people so, consider as 3 ary tree</p> <p>total no. of nodes=3*internal nodes+1</p> $100=3*i+1$ $i=33$ <p>Thus, terminal nodes are $100-33= 67$</p> <p>Therefore 67 person do not make any call</p> <p>As each call take 5 min.</p> <p>after 5 min 1+3 people receive the message</p> <p>after 10 min 1+3+9=13 people receive the message</p> <p>after 15 min 1+3+9+27=40 person getting the message</p> <p>after 20 min 1+3+9+27+81=121 person could get the message</p>	
C	Construct a unique binary tree whose in-order, pre-order and post-order traversal is given below:	4
	<p>a) In-order : d g b e i h j a c f</p> <p>Pre-order: a b d g e h i j c f</p> <p>b) In-order : d c e b f a h g i</p> <p>Post-order: d e c f b h i g a</p>	
ANS		



D. Consider the poset $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ under partial order whose Hasse diagram is given below. Consider the subset $B = \{1, 2\}$, $C = \{3, 4, 5\}$ of A .

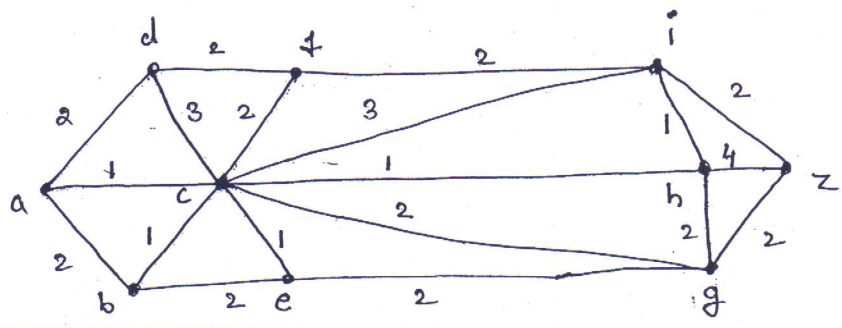


Determine:
 a) All the Lower bound and Upper bound of B, C .
 b) $glb(B)$ & $lub(B)$
 c) $glb(C)$ & $lub(C)$

ANS $UB(B) = 3, 4, 5, 6, 7, 8$ $UB(C) = 6, 7, 8$
 $LB(B) = \text{none}$ $LB(C) = 1, 2, 3$
 $glb(B) = \text{none}$ $lub(B) = 3$
 $glb(C) = 3$ $lub(C) = \text{none}$

Qu 2.

A. Find the shortest path between a and z for the following graph using Dijkstra's algorithm.



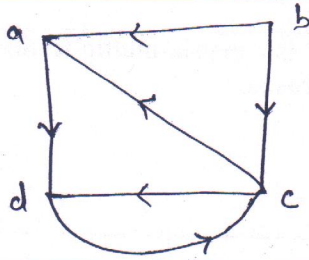
ANS Minimum distance is 5

4

5

	Shortest path is $P = \{a, c, h, i, z\}$	
B.	Find the fundamental cut set for the graph G with respect to spanning tree T is given below:	3
	<p>The diagram shows a graph G with 8 vertices (v_1 to v_8) and 13 edges (e_1 to e_{13}). A spanning tree T is also shown, consisting of 7 edges ($e_2, e_4, e_5, e_7, e_{10}, e_{12}, e_{13}$).</p>	
ANS	$e_2 \{e_1, e_2, e_3\}$ $e_4 \{e_1, e_4, e_6\}$ $e_5 \{e_3, e_5, e_8\}$ $e_7 \{e_6, e_7, e_8\}$ $e_{10} \{e_6, e_9, e_{10}, e_{11}\}$ $e_{12} \{e_9, e_{12}\}$ $e_{13} \{e_{11}, e_{13}\}$	
C.	When a wheel graph on n vertices is regular? Draw the appropriate graph.	2
ANS	$n=3$	
D.	Show that if 20 persons are selected for presenting a cultural programme, then one may select a subset of 3 so that all three would be able to present their programmes on the same day of the week.	3
ANS	$(n-1)/m + 1$ where $m=7, n=20$ $(20-1)/7 + 1 = 3$ of the person must present their programmes on the same day of the week	
Qu3.		
A.	Which of the following degree sequence represent a simple non directed graph? Justify your answer with appropriate reason.	2
	a) $\{2, 3, 3, 4, 4, 5\}$ b) $\{2, 3, 4, 4, 5\}$ c) $\{1, 3, 3, 4, 5, 6, 6\}$ d) $\{2, 2, 3, 3\}$	
ANS	d	
B.	Find how many integers from 1 to 60 that are divisible by 2 nor by 3 and nor by 5. Also determine the number of integers divisible by 5, not by 2, not by 3.	3
ANS	$a = 60/2 = 30$ $B = 60/3 = 20$ $C = 60/5 = 12$ $A \cap B = 60/6 = 10$ $A \cap C = 60/10 = 6$ $B \cap C = 60/15 = 4$ $A \cap B \cap C = 60/30 = 2$	

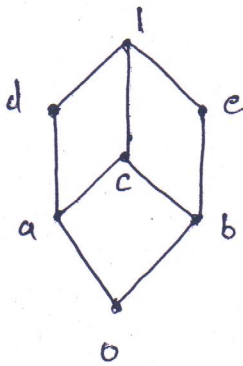
	$A'IB'IC' = 60 - 30 - 20 - 12 + 10 + 6 + 4 - 2 = 16$ $(AIC) - (AIBIC) = 6 - 2 = 4$ $(BIC) - (AIBIC) = 4 - 2 = 2$ Therefore integer divisible by 5 but not by 2 and 3 = $12 - 4 - 2 = 4$	
C.	Check the validity of the following arguments: a) "if A has completed MCA or MBA, then she is assured of a good job. If A is assured of a good job, she is happy. A is not happy. So A has not completed MBA". b) "If today is Sunday, then Yesterday was Saturday. Yesterday was Saturday. Today is Sunday". c) "If I drive to work then I will arrive in time. I do not drive to work. Therefore I will not arrive in time". d) "If I try hard and I have a talent, then I will become Scientist. If I become a Scientist, then I will be happy. Therefore, if I will not be happy, then I did not try hard or do not have talent".	2
ANS	a. valid b. not valid c. not valid d. Valid	<input type="radio"/>
D.	Let $R = \left\{ \begin{bmatrix} a & b \\ b & a \end{bmatrix} : a, b \in \mathbb{Z} \right\}$ f is the mapping that takes $\begin{bmatrix} a & b \\ b & a \end{bmatrix}$ to $(a - b)$ Show that it is a Homomorphism?	3
	$f\left(\begin{bmatrix} a & b \\ b & a \end{bmatrix} + \begin{bmatrix} c & d \\ d & c \end{bmatrix}\right) = f\left(\begin{bmatrix} a+c & b+d \\ b+d & a+c \end{bmatrix}\right) = a+c - (b+d)$ $= (a-b) + (c-d)$ $= f\left(\begin{bmatrix} a & b \\ b & a \end{bmatrix}\right) + f\left(\begin{bmatrix} c & d \\ d & c \end{bmatrix}\right)$ $f\left(\begin{bmatrix} a & b \\ b & a \end{bmatrix} \cdot \begin{bmatrix} c & d \\ d & c \end{bmatrix}\right) = f\left(\begin{bmatrix} ac+bd & ad+bc \\ bct+ad & bd+ac \end{bmatrix}\right)$ $= (ac+bd) - (ad+bc)$ $= (a-b)(c-d) \Rightarrow f\left(\begin{bmatrix} a & b \\ b & a \end{bmatrix}\right) \cdot f\left(\begin{bmatrix} c & d \\ d & c \end{bmatrix}\right)$	<input type="radio"/>
Qu4.		
A.	Find the matrix of transitive closure of R using Warshall's algorithm.	3



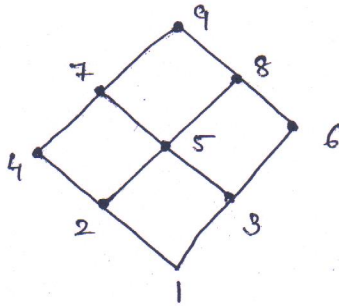
ANS $P(4) =$
 1 0 1 1
 1 0 1 1
 1 0 1 1
 1 0 1 1

B Which of the following posets are lattices? Justify your answer with appropriate reason.

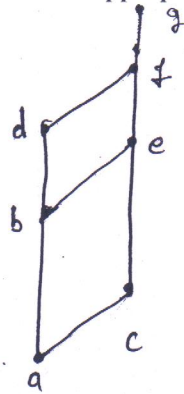
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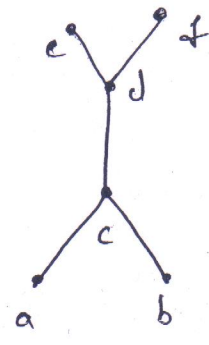
(a)



(b)



(c)



(d)

ANS a) Y b) Y c) Y d) N

C. Solve $a_r - 5a_{r-1} + 6a_{r-2} = 2^r + r$, $r \geq 2$ with $a_0 = a_1 = 1$

3

ANS $a_r = 2^r(-5/4 - 2^r) + 7/4(3^r + r) + 1/2$

D. Consider the following Statements:

2

$$S1: \exists x (P(x) \rightarrow Q(x)) \Leftrightarrow \forall x P(x) \rightarrow \exists x Q(x)$$

$$S2: [\exists x P(x) \rightarrow \forall x Q(x)] \Leftrightarrow \forall x [P(x) \rightarrow Q(x)]$$

Which of the statement is correct?

- a) S1 is True & S2 is False
- b) S1 is False & S2 is True
- c) S1 is True & S2 is True
- d) S1 is False & S2 is False

ANS	a	
E	If A is a set of all non singular matrices of order n and $*$ is a matrix multiplication operation then which of the following is false? Justify with appropriate reason. a) $(A, *)$ is a monoid b) $(A, *)$ is a group c) $(A, *)$ is a semigroup d) $(A, *)$ is a abelian	2
ANS	d	