Math

College of Engineering, Pune
(An Autonomus Institute of Government of Maharashtra, Pune-411005)
End-Semester Exam November 2012
(MA 201) Engineering Mathematics III

Programme : S.Y.B.Tech. | Brancehs : All
Academic Year : 2012-13 | Date : 23/11/2012
Duration: 10.00 am to 1.00 pm | Max. Marks : 50

Instructions:
(1) All questions are compulsory. (2) All symbols have their usual meanings. (3) Figures to the right indicate maximum marks. (4) **Begin a new question on new page and solve all the subquestions together.** (5) If you use any notations other than standard notations, describe them specifically so that your answers become self explanatory.(6) The required statistical tables are provided in the question paper.

Q.1]

(A) Determine whether the following statements are **True** or **False**. Justify your answer.
(i) Let $T(x, y, z)$ be a differentiable scalar function that represents a surface $S : T(x, y, z) = c$ and gradient of T at point P on S is not zero. Then there is no change in the value of T at P in the direction of a tangent to any curve on S through point P.
(ii) The periodic function $f(x) = |\cos 5x|$ where $-\pi/10 < x < \pi/5$ is an even function. [3]

(B) Find the Fourier series of

$$f(x) = \begin{cases} 1 + (2x/\pi) & \text{if} \quad (-\pi \le x \le 0); \\ 1 - (2x/\pi) & \text{if} \quad (0 \le x \le \pi) \end{cases}$$

Hence show that $1 + \frac{1}{9} + \frac{1}{25} + \cdots = \frac{\pi^2}{8}$ [3]

(C) Using $\operatorname{div}(f\overline{v}) = f\operatorname{div}\,\overline{v} + \overline{v}\cdot\nabla f$ and $\nabla(f^n) = nf^{n-1}\nabla f$,
find div $\overline{v}$, where $\overline{v} = (x^2 + y^2 + z^2)^{-3/2}(x\mathbf{i} + y\mathbf{j} + z\mathbf{k})$. [2]

(D) Show that the vector field $\overline{v} = [2xyz^2, x^2z^2 + z\cos(yz), 2x^2yz + y\cos(yz)]$ is conservative. Determine the corresponding scalar potential. [2]

Q.2]

(A) For **any two** following surfaces, give a parametric representation with limits of parameters.
(i) Portion of plane $3x + 4y + 6z = 24$ in the first octant.
(ii) Parabolic cylinder $z = 4x^2, 0 \le y \le 3, x \ge 0$
(iii) Cone $z = 2\sqrt{x^2 + y^2},\ 0 \le z \le 1, x \le 0$ [2]

(B) Find the moment of inertia (M.I.) about z axis of the curved surface of cylinder $x^2+y^2 = 1, 0 \le z \le h$ $y \ge 0$ with density $\delta = yz$.
Note: M.I. about $L = \int\int_S r^2\delta\ dA$ where r is the perpendicular distance from point (x,y,z) to axis L. [3]

(C) (i) State Stokes's Theorem and prove Green's theorem as a special case of Stokes's Theorem. [2]
(ii) Evaluate $\int\int_S \overline{F}\cdot\hat{n}\ dA$ where $\bar{F} = [x^3, y^3, z^3]$; S : the surface of the sphere $x^2 + y^2 + z^2 = 4$. [3]

OR

(C) Find the flux across the curved surface of the paraboloid $z = x^2 + y^2,\ 0 \le z \le 4, x \ge 0$ when the velocity vector is $\bar{V} = [x, y, z]$ [5]

Q.3]

(A) Is $L(f(at + b)) = F(as + b)$ where $F(s) = L(f(t))$ true? Justify your answer. [2]

(B) Solve the initial value problem by using the Laplace Transforms. Show all the details.
$y'' + 2y' + 5y = 50t - 150;\ \ y(3) = -4, y'(3) = 14$ [4]

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(C) Attempt any two: [4]

(i) Find inverse laplace transform of $\dfrac{2}{(s^2+6s+10)^2} - 2$

(ii) Prove the following Rule -

$$L(f(t)/t) = \int_s^{\infty} F(\tilde{s})d(\tilde{s}) \text{ where } F(s) = L(f(t))$$

(iii) Using Laplace transform, find the current in a circuit containing series combination of inductor, resistor and capacitor with values - $R = 2$ ohm, $L = 1H, C = 0.5F, i(0) = 0, q(0) = 0$. $v = 1kV$ if $0 < t < 2$ and 0 if $t > 2$.

Q.4]

(A) Let a continuous random variable X have density function $f(x)$ which is symmetric about the mean μ. Then the k^{th} central moment of X is defined as

$$E((X-\mu)^k) = \int_{-\infty}^{\infty} (x-\mu)^k f(x)dx.$$

Prove that k^{th} central moment is zero if k is odd. [2]

(B) An important factor in solid missile fuel is the particle size distribution. Significant problems occur if the particle sizes are too large. From production data in the past, it has been determined that the particle size (in micrometers) distribution is characterized by

$$f(x) = \begin{cases} 3x^{-4} & \text{if} \quad (x > 1); \\ 0 & \text{elsewhere} \end{cases}$$

(i) Verify that this is a valid density function.

(ii) Evaluate $F(x)$ and hence find the probability that a random particle from the manufactured fuel exceeds 4 micrometers?

[2]

(C) Five is the average number of oil tankers arriving each day at a certain port city. The facilities at the port can handle at most 8 tankers per day. What is the probability that on a given day tankers have to be turned away? [1]

(D) A soft drink machine is regulated so that it discharges an average of 200 ml. cup. If the amount of drink is normally distributed with a standard deviation equal to 15 ml.,

(i) How many cups will probably overflow if 230 ml. cups are used for the next 1000 drinks? [1.5]

(ii) Below what value do we get the smallest 25% of the drinks? [1.5]

(iii) Test the hypothesis $\mu = 200$ against $\mu \neq 200$ if it is observed that a random sample of 50 cups contain 201 ml on an average. Use 5% level of significance. [2]

OR

(D) The faces of a thin square copper plate of side 24 cm. are perfectly insulated. The right side is kept at 20 ^0C and the other sides are kept at 0 ^0C. Find the steady state temperature $u(x, y)$ in the plate. Show all the details. [5]

Q.5]

(A) (i) State the general solution of one dimensional wave equation $\dfrac{\partial^2 u}{\partial t^2} = c^2 \dfrac{\partial^2 u}{\partial x^2}$ subject to boundary conditions $u(0,t) = u(L,t) = 0$ for all t. [1]

(ii) Using solution in (i), find the deflection $u(x,t)$ of the string of length $L = \pi$ when $c = 1$, if the initial velocity is zero, and the initial deflection is

$$f(x) = \begin{cases} \frac{x}{\pi} & \text{if} \quad (0 \le x \le \pi/2); \\ 1 - \frac{x}{\pi} & \text{if} \quad (\pi/2 \le x \le \pi) \end{cases}$$

[2]

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(iii) Express $u(x,t)$ as superposition of two functions (show details) and also sketch $u(x,t)$ for $t = 2\pi/5$ [

OR

(A) (i) State the general solution of two dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2})$ subject to boundary conditions $u = 0$ on boundaries of rectangular membrane. [1]

(ii) Using solution in (i), find the deflection $u(x,y,t)$ of the square membrane with $c = 1$, if initial velocity is zero and the initial deflection is given by

$f(x,y) = kxy(1-x)(1-y); a = b = 1$ [2]

(iii) Define nodal lines and sketch them for the eigen functions u_{12} and u_{31}. [2]

(B) Ends A and B of a bar of length 10 cm are kept at 20 ^0C and 40 ^0C respectively until steady state conditions prevail. The temperature at A is then suddenly raised to 50 ^0C and at the same time at B lowered to 10 ^0C and these are maintained. Find the subsequent temperature distribution. [5]

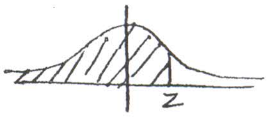
OR

(B) Obtain the temperature $u(x,t)$ at any point x and at any time $t > 0$ in a thin long bar of length L where the heat flow is governed by the PDE $\frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial x^2}$ if both the ends are insulated and initial temp. is given by $u(x,0) = f(x)$. [5]

Poisson Distribution

x	$\mu = 4$ $f(x)$	$F(x)$	$\mu = 5$ $f(x)$	$F(x)$
	0.		0.	
0	0183	0.0183	0067	0.0067
1	0733	0.0916	0337	0.0404
2	1465	0.2381	0842	0.1247
3	1954	0.4335	1404	0.2650
4	1954	0.6288	1755	0.4405
5	1563	0.7851	1755	0.6160
6	1042	0.8893	1462	0.7622
7	0595	0.9489	1044	0.8666
8	0298	0.9786	0653	0.9319
9	0132	0.9919	0363	0.9682
10	0053	0.9972	0181	0.9863
11	0019	0.9991	0082	0.9945
12	0006	0.9997	0034	0.9980
13	0002	0.9999	0013	0.9993
14	0001	1.0000	0005	0.9998
15			0002	0.9999
16			0000	1.0000

Normal Distribution



z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$
	0.		0.		0.		0.		0.		0.
0.01	5040	0.51	6950	1.01	8438	1.51	9345	2.01	9778	2.51	9940
0.02	5080	0.52	6985	1.02	8461	1.52	9357	2.02	9783	2.52	9941
0.03	5120	0.53	7019	1.03	8485	1.53	9370	2.03	9788	2.53	9943
0.04	5160	0.54	7054	1.04	8508	1.54	9382	2.04	9793	2.54	9945
0.05	5199	0.55	7088	1.05	8531	1.55	9394	2.05	9798	2.55	9946
0.06	5239	0.56	7123	1.06	8554	1.56	9406	2.06	9803	2.56	9948
0.07	5279	0.57	7157	1.07	8577	1.57	9418	2.07	9808	2.57	9949
0.08	5319	0.58	7190	1.08	8599	1.58	9429	2.08	9812	2.58	9951
0.09	5359	0.59	7224	1.09	8621	1.59	9441	2.09	9817	2.59	9952
0.10	5398	0.60	7257	1.10	8643	1.60	9452	2.10	9821	2.60	9953
0.11	5438	0.61	7291	1.11	8665	1.61	9463	2.11	9826	2.61	9955
0.12	5478	0.62	7324	1.12	8686	1.62	9474	2.12	9830	2.62	9956
0.13	5517	0.63	7357	1.13	8708	1.63	9484	2.13	9834	2.63	9957
0.14	5557	0.64	7389	1.14	8729	1.64	9495	2.14	9838	2.64	9959
0.15	5596	0.65	7422	1.15	8749	1.65	9505	2.15	9842	2.65	9960
0.16	5636	0.66	7454	1.16	8770	1.66	9515	2.16	9846	2.66	9961
0.17	5675	0.67	7486	1.17	8790	1.67	9525	2.17	9850	2.67	9962
0.18	5714	0.68	7517	1.18	8810	1.68	9535	2.18	9854	2.68	9963
0.19	5753	0.69	7549	1.19	8830	1.69	9545	2.19	9857	2.69	9964
0.20	5793	0.70	7580	1.20	8849	1.70	9554	2.20	9861	2.70	9965

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Table A8
Normal Distribution

Values of z for given values of $\Phi(z)$ [see (3), Sec. 22.8] and $D(z) = \Phi(z) - \Phi(-z)$
Example: $z = 0.279$ if $\Phi(z) = 61\%$; $z = 0.860$ if $D(z) = 61\%$.

%	$z(\Phi)$	$z(D)$	%	$z(\Phi)$	$z(D)$	%	$z(\Phi)$	$z(D)$
1	−2.326	0.013	41	−0.228	0.539	81	0.878	1.311
2	−2.054	0.025	42	−0.202	0.553	82	0.915	1.341
3	−1.881	0.038	43	−0.176	0.568	83	0.954	1.372
4	−1.751	0.050	44	−0.151	0.583	84	0.994	1.405
5	−1.645	0.063	45	−0.126	0.598	85	1.036	1.440
6	−1.555	0.075	46	−0.100	0.613	86	1.080	1.476
7	−1.476	0.088	47	−0.075	0.628	87	1.126	1.514
8	−1.405	0.100	48	−0.050	0.643	88	1.175	1.555
9	−1.341	0.113	49	−0.025	0.659	89	1.227	1.598
10	−1.282	0.126	50	0.000	0.674	90	1.282	1.645
11	−1.227	0.138	51	0.025	0.690	91	1.341	1.695
12	−1.175	0.151	52	0.050	0.706	92	1.405	1.751
13	−1.126	0.164	53	0.075	0.722	93	1.476	1.812
14	−1.080	0.176	54	0.100	0.739	94	1.555	1.881
15	−1.036	0.189	55	0.126	0.755	95	1.645	1.960
16	−0.994	0.202	56	0.151	0.772	96	1.751	2.054
17	−0.954	0.215	57	0.176	0.789	97	1.881	2.170
18	−0.915	0.228	58	0.202	0.806	97.5	1.960	2.241
19	−0.878	0.240	59	0.228	0.824	98	2.054	2.326
20	−0.842	0.253	60	0.253	0.842	99	2.326	2.576
21	−0.806	0.266	61	0.279	0.860	99.1	2.366	2.612
22	−0.772	0.279	62	0.305	0.878	99.2	2.409	2.652
23	−0.739	0.292	63	0.332	0.896	99.3	2.457	2.697
24	−0.706	0.305	64	0.358	0.915	99.4	2.512	2.748
25	−0.674	0.319	65	0.385	0.935	99.5	2.576	2.807
26	−0.643	0.332	66	0.412	0.954	99.6	2.652	2.878
27	−0.613	0.345	67	0.440	0.974	99.7	2.748	2.968
28	−0.583	0.358	68	0.468	0.994	99.8	2.878	3.090
29	−0.553	0.372	69	0.496	1.015	99.9	3.090	3.291
30	−0.524	0.385	70	0.524	1.036			
31	−0.496	0.399	71	0.553	1.058	99.91	3.121	3.320
32	−0.468	0.412	72	0.583	1.080	99.92	3.156	3.353
33	−0.440	0.426	73	0.613	1.103	99.93	3.195	3.390
34	−0.412	0.440	74	0.643	1.126	99.94	3.239	3.432
35	−0.385	0.454	75	0.674	1.150	99.95	3.291	3.481
36	−0.358	0.468	76	0.706	1.175	99.96	3.353	3.540
37	−0.332	0.482	77	0.739	1.200	99.97	3.432	3.615
38	−0.305	0.496	78	0.772	1.227	99.98	3.540	3.719
39	−0.279	0.510	79	0.806	1.254	99.99	3.719	3.891
40	−0.253	0.524	80	0.842	1.282			