College of Engineering, Pune - 411005.

(MA-201) Engineering Mathematics - III

(Academic Year: 2013-14)

End Semester Examination

Date: November , 2013 ♣ Max. Marks: 60 ♣ Duration: 3 hours.

Instructions: 1) All questions are compulsory. 2) Figures to the right indicate full marks. 3) Use of non-programmable calculator is allowed.

Q.1 Attempt ALL the following: [12]

(a) State whether True/False. Justify your answer.
curl $(\overline{u} + \overline{v}) = \text{curl}\,\overline{u} + \text{curl}\,\overline{v}$, for any vector fields $\overline{u}, \overline{v}$.

(b) Show that the work done by a constant force field $\overline{F} = [a, b, c]$ in moving a particle along any path from point A to B is $W = \overline{F} \cdot \overline{AB}$.

(c) Find the solution $u(x, y)$ of the p.d.e. $y^3 u_x + x^2 u_y = 0$ by the method of separation of variables.

(d) Find the Laplace transform of $f(t) = \cos^2(\pi t)$ using the Laplace transform of first derivative of $f(t)$.

(e) If $g(t)$ is a continuous function, then prove that $\int_0^\infty g(t)\, \delta(t - a)\, dt = g(a)$.

(f) A box contains 100 transistors, 20 of which are defective. If 10 transistors are selected at random, what is the probability that all 10 will be defective?

Q.2 (a) Find the surface area of the part of the plane $y = x$ included in the first octant, bounded by $y = 0, y = 1, z = 0, z = 2$. [2]

(b) Let $\overline{r} = xi + yj + zk$ and $r = |\overline{r}|$. Find $\nabla \cdot \left(\frac{\overline{r}}{r}\right)$. [2]

(c) Find the directional derivative of $f(x, y, z) = x^2yz + 4xz^2$ at $P(1, 2, -1)$ along the line $\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z-3}{-2}$. [4]

(d) Evaluate $\iint_S (\nabla \times \overline{F}) \cdot \hat{n}\, dA$ using Stoke's theorem, where $\overline{F} = [y^2, z^2, x^2]$ and $S : x^2 + y^2 = z,\ y \geq 0,\ z \leq 1$. [4]

OR

Evaluate $\oint_C \overline{F} \cdot d\overline{r}$, where $\overline{F} = [y^2, x^2, -x + z]$ and C is the boundary of the triangle with vertices (0,0,1), (1,0,1) and (1,1,1). [4]

Q.3 (a) Let $f(x) = 4 - x^2, 0 < x < 2$ be a periodic function with period 2. Find the sum of the Fourier series of $f(x)$ at $x = 1$ and $x = 2$. [2]

(b) Derive the Euler's formulas for the Fourier coefficients of a 2π-periodic even function $f(x)$ defined in the interval $(-\pi, \pi)$. [2]

(c) Derive one-dimensional wave equation which governs the transverse vibrations of an elastic string of length L. [4]

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(d) Find the inverse Laplace transform of [4]

$$e^{-s}\left[s\,\ln\left(\frac{s}{\sqrt{s^2+1}}\right)+\cot^{-1}s\right]. \quad OR \quad \frac{1}{s}\left[\ln\sqrt{a-\frac{a^2}{s^2}}\right]$$

Q.4 (a) Rewrite $f(t)$ using unit step function and then find its Laplace transform, where

$$f(t)=\begin{cases}\cos t, & 0<t<\pi,\\ 0, & \pi<t<2\pi,\\ \sin 2t, & t>2\pi.\end{cases}$$ [2]

(b) Find the Laplace transform of $\dfrac{e^{-3t}\,\delta(t-1)}{t}$. [2]

(c) Find the inverse Laplace transform of $\dfrac{1}{(s^2+4s+8)^2}$. [4]

(d) Attempt ANY ONE:

(1) Solve using Laplace transform: $y''-4y'+3y=6t-8,\ y(0)=y'(0)=0$. [4]

(2) (i) Solve the integral equation: $y(t)=\sin 2t+\int_0^t y(\tau)\,\sin(t-\tau)\,d\tau$. [2]

(ii) Evaluate $\int_0^\infty \dfrac{e^{-3t}\,\sin t}{t}\,dt$, using properties of Laplace transform. [2]

Q.5 Attempt ANY THREE: [12]

(a) i. Find the probability of getting 3 blue marbles if 5 marbles are drawn one after the other without replacement from a box containing 6 blue and 4 red marbles.

ii. Let X be normal with mean 10 and variance 4. Find $P[11.1<X<14.02]$.

(b) A manufacturer of office files knows that 1 % of his product is defective. He sells the files in boxes of 100 files each and guarantees that a box contains at most 2 defective files. Find the probability that a box will meet the guarantee.

(c) If 10 % of the students receive 'AA' grade and 5 % of the students receive 'FF' grade in an examination, determine the
(1) minimum marks for getting 'AA',
(2) minimum marks to pass the examination (i.e. minimum marks required *not* to get 'FF' grade), assuming that the marks are normally distributed with mean 66 and standard deviation 15.

(d) In order to start new S.T. bus to a certain remote village, it is required to get the average fare of Rs.40000 daily. Reports on number of passengers for 21 days revealed that the average daily collection of fare was Rs.39000, with standard deviation of Rs.4000. Do these data support the demand of the people for starting new bus to the village? Assume normality and use 5 % level of significance.

Statistical Tables

P.T.O.

Poisson Distribution

Probability function $f(x)$, Distribution function $F(x)$.

x	$\mu = 0.6$ $f(x)$	$F(x)$	$\mu = 0.7$ $f(x)$	$F(x)$	$\mu = 0.8$ $f(x)$	$F(x)$	$\mu = 0.9$ $f(x)$	$F(x)$	$\mu = 1$ $f(x)$	$F(x)$
	0.		0.		0.		0.		0.	
0	5488	0.5488	4966	0.4966	4493	0.4493	4066	0.4066	3679	0.3679
1	3293	0.8781	3476	0.8442	3595	0.8088	3659	0.7725	3679	0.7358
2	0988	0.9769	1217	0.9659	1438	0.9526	1647	0.9371	1839	0.9197
3	0198	0.9966	0284	0.9942	0383	0.9909	0494	0.9865	0613	0.9810
4	0030	0.9996	0050	0.9992	0077	0.9986	0111	0.9977	0153	0.9963
5	0004	1.0000	0007	0.9999	0012	0.9998	0020	0.9997	0031	0.9994
6			0001	1.0000	0002	1.0000	0003	1.0000	0005	0.9999
7									0001	1.0000

Table A7 Normal Distribution

Values of the distribution function $\Phi(z)$ [see (3), Sec. 24.8]. $\Phi(-z) = 1 - \Phi(z)$

z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$
	0.		0.		0.		0.		0.		0.
0.01	5040	0.51	6950	1.01	8438	1.51	9345	2.01	9778	2.51	9940
0.02	5080	0.52	6985	1.02	8461	1.52	9357	2.02	9783	2.52	9941
0.03	5120	0.53	7019	1.03	8485	1.53	9370	2.03	9788	2.53	9943
0.04	5160	0.54	7054	1.04	8508	1.54	9382	2.04	9793	2.54	9945
0.05	5199	0.55	7088	1.05	8531	1.55	9394	2.05	9798	2.55	9946
0.06	5239	0.56	7123	1.06	8554	1.56	9406	2.06	9803	2.56	9948
0.07	5279	0.57	7157	1.07	8577	1.57	9418	2.07	9808	2.57	9949
0.08	5319	0.58	7190	1.08	8599	1.58	9429	2.08	9812	2.58	9951
0.09	5359	0.59	7224	1.09	8621	1.59	9441	2.09	9817	2.59	9952
0.10	5398	0.60	7257	1.10	8643	1.60	9452	2.10	9821	2.60	9953

Table A8 Normal Distribution

Values of z for given values of $\Phi(z)$ [see (3), Sec. 24.8] and $D(z) = \Phi(z) - \Phi(-z)$
Example: $z = 0.279$ if $\Phi(z) = 61\%$; $z = 0.860$ if $D(z) = 61\%$.

%	$z(\Phi)$	$z(D)$	%	$z(\Phi)$	$z(D)$	%	$z(\Phi)$	$z(D)$
1	−2.326	0.013	41	−0.228	0.539	81	0.878	1.311
2	−2.054	0.025	42	−0.202	0.553	82	0.915	1.341
3	−1.881	0.038	43	−0.176	0.568	83	0.954	1.372
4	−1.751	0.050	44	−0.151	0.583	84	0.994	1.405
5	−1.645	0.063	45	−0.126	0.598	85	1.036	1.440
6	−1.555	0.075	46	−0.100	0.613	86	1.080	1.476
7	−1.476	0.088	47	−0.075	0.628	87	1.126	1.514
8	−1.405	0.100	48	−0.050	0.643	88	1.175	1.555
9	−1.341	0.113	49	−0.025	0.659	89	1.227	1.598
10	−1.282	0.126	50	0.000	0.674	90	1.282	1.645
11	−1.227	0.138	51	0.025	0.690	91	1.341	1.695
12	−1.175	0.151	52	0.050	0.706	92	1.405	1.751
13	−1.126	0.164	53	0.075	0.722	93	1.476	1.812
14	−1.080	0.176	54	0.100	0.739	94	1.555	1.881
15	−1.036	0.189	55	0.126	0.755	95	1.645	1.960