COLLEGE OF ENGINEERING, PUNE

END SEMESTER EXAMINATION NOV/ DEC, 2012

T.Y.B.Tech (ET301) RANDOM SIGNALS AND STOCHASTIC PROCESSES

Duration -3 Hours **Max Marks- 50**

Instructions to candidates:

1. All questions are compulsory.
2. Assume suitable data if required.
3. Figures to the right indicate maximum marks.
4. Non programmable scientific calculators are allowed.

Q.1. a.	A lab blood test is 95% effective in detecting a certain disease when it is in fact, present. However, the test also yields a false positive result for 1% of the healthy persons tested (i.e., if a healthy person is tested, then with probability 0.01, the test result will imply he has the disease.) If 0.5% of the population actually has the disease, what is the probability has the disease given that his test result is positive?	[2]
b.	A random variable x has density function given by $f(x)= 2 e^{-2x} \quad x \geq 0$ $= 0 \quad x < 0$ Find the moment generating function and first four moments about the origin.	[4]
c.	The joint density function of two continuous random variable X and Y is $f(x, y) = cxy \quad 0 < x < 4, 1 < y < 5$ $= 0 \quad$ otherwise Find $P(1 < X < 2, 2 < Y < 3)$	[4]
Q.2.	Find the characteristics function of the random variable having density function $f(x) = c\, e^{-a\lvert x \rvert} \qquad -\infty < x < \infty,$ Where a>0 and c is suitable constant.	[5]
Q.3	x and y are independent, identically distributed random variables with common pdf $fx(x) = e^{-x} u(x), \qquad fy(y) = e^{-y} u(y).$ Find the pdf of the following random variables 1. $x + y$ 2. $x - y$	[5]
Q.4	Narrow band noise representation in terms of low frequency sums (x(t) and y(t)) given by $n(t) = x(t)\cos \omega_0 t - y(t)\sin w_0 t$ Show that $E(x^2) = E(y^2) = N$ Similarly Show that $E(x) = E(y) = 0$ and $E(xy) = 0$, as well.	[5]
	OR	
	An FSK system transmits $2 * 10^6$ bits/ sec. White Gaussian noise is added during transmission. The amplitude of either signal at the receiver input is 0.45μV, while the white noise spectral density at the same point is $n_0 / 2 = 1 / 2 * 10^{-20}$ volts^2 / Hz. Compare the probability of error for a receiver using synchronous detection with one using envelope detection. Assume matched filter detection in both cases.	[5]

Q.5	Consider the random process $z(t) = x \cos(\omega_0 t + \theta) - y \sin(\omega_0 t + \theta)$ where x and y are independent random variable with zero expected value and variance σ^2; θ a uniformly distributed $(0, 2\pi)$ random variable independent of x and y. Show that z (t) is also normal (Gaussian) with zero expected value and variance σ^2.	[5]
Q.6	What is the probability that the envelope of narrowband Gaussian noise will exceed three times its rms value?	[5]
	OR	
	White noise of spectral density $G_n(f) = n_0 / 2$ watts/ Hz is applied to an ideal low pass filter of bandwidth B Hz and transfer amplitude A. Find the correlation function of noise at the output. Calculate the total average power at the filter output from the spectral density, and compare with R (0).	[5]
Q.7	A miner is trapped in a mine containing three doors. The first door leads to a tunnel that takes him to safety after two hours of travel. The second door leads to a tunnel that returns him to the mine after three hours of travel. The third door leads to a tunnel that returns him mine after five hours. Assuming that the miner is at all times equally likely to choose any one of the doors, what is the expected length of time until the miner reaches safety?	[5]
Q.8	A simple radar system is to be designed, using threshold detection criterion. The probability that noise alone will be mistaken for signal is specified at 10^{-10}. What must the average SNR at the input to the envelope detector be to keep the probability of missing the signal pulse, when it appears, to 0.1? Assume that the SNR and threshold level are high enough so that the Gaussian density function approximated to the distribution of signal plus noise may be used. Verify this.	[5]
Q.9.a	Define WSS random sequence. Show that all stationary random sequences are WSS.	[3]
b.	State and prove Chebyshev's inequality.	[2]