College of Engineering, Pune - 411005.

# LINEAR ALGEBRA

(Academic Year: 2013-14)

## End Semester Examination

Date: November 21, 2013 ♣ Max. Marks: 60 ♣ Time: 2 p.m. - 5 p.m.

**Instructions:** 1) All questions are compulsory. 2) Figures to the right indicate full marks. 3) Use of non-programmable calculator is allowed.

1. Solve the following system by Doolittle's method (LU decomposition): [5]

$$\begin{aligned} 5x_1 + 4x_2 + x_3 &= 3.4 \\ 10x_1 + 9x_2 + 4x_3 &= 8.8 \\ 10x_1 + 13x_2 + 15x_3 &= 19.2 \end{aligned}$$

2. Solve the following system by Gauss-Seidel method (upto 4 iterations) starting with initial approximation (1,1,1). [6]

$$\begin{aligned} 5x_1 + x_2 + 2x_3 &= 19 \\ x_1 + 4x_2 - 2x_3 &= -2 \\ 2x_1 + 3x_2 + 8x_3 &= 39 \end{aligned}$$

3. Fit a parabola to the given points $(x, y)$ by the method of least squares. [5]

| | | | | | |
|---|---|---|---|---|---|
| Speed of a snowplow $x[mph]$ | 5 | 7.5 | 10 | 12.5 | 15 |
| Power of the plow $y[lb]$ | 4200 | 4600 | 5200 | 4800 | 4300 |

4. Find the largest eigenvalue ($\lambda$), the corresponding eigenvector and the error $\epsilon$ of $\lambda$ for the following matrix by Power method. Start with $x_0 = [1\,1\,1]^t$ and upto $x_6$. [6]

$$\begin{bmatrix} 0.49 & 0.02 & 0.22 \\ 0.02 & 0.28 & 0.20 \\ 0.22 & 0.20 & 0.40 \end{bmatrix}$$

5. Tridiagonalize the matrix $\begin{bmatrix} 7 & 2 & 3 \\ 2 & 10 & 6 \\ 3 & 6 & 7 \end{bmatrix}$ by Householder's method. [4]

6. Find the minimal polynomial of $\begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$. [3]

7. Suppose the characteristic and minimal polynomials of an operator $T$ are $ch(T) = (\lambda - 2)^4(\lambda - 5)^3$ and $m(T) = (\lambda - 2)^2(\lambda - 5)^3$ respectively. Determine all possible Jordan canonical forms of $T$. [4]

8. Find the characteristic and minimal polynomial of $\begin{bmatrix} 1 & -6 & 0 & 0 & 0 \\ -0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 4 & 2 & 0 \\ 0 & 0 & 3 & 1 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$. [5]



P.T.O. →

9. Classify the following matrices as Hermitian/skew-Hermitian/Unitary. [5]

$$\begin{bmatrix} 4i & 0 & i \\ 0 & i & 0 \\ i & 0 & 4i \end{bmatrix}, \begin{bmatrix} i & 0 & 0 \\ 0 & 0 & i \\ 0 & i & 0 \end{bmatrix}.$$

10. State and prove rank-nullity theorem.

*OR*

Let $V$ be a vector space which is spanned by a finite set of vectors $v_1, v_2, \ldots, v_m$. Prove that any linearly independent set in $V$ contains no more than $m$ vectors. [6]

11. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation whose matrix relative to the standard ordered basis is $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$. Find a basis for the null space of $T$. [6]

12. Consider $\mathbb{R}^4$ with standard inner product. Let $W$ be the subspace of $\mathbb{R}^4$ consisting of all vectors which are orthogonal to both (1,0,-1,1) and (2,3,-1,2).
Find a basis for $W$. [5]