College of Engineering, Pune
(An Autonomous Institute of Government of Maharashtra, Pune-411005)
End-Semester Exam Nov 2013
MA 203 Foundations of Mathematics I

Programme : S.Y.B.Tech. Brancehs : All (Diploma Students)
Academic Year : 2013-14 Date : /11/2012
Duration: 3 Hrs. Max. Marks : 60

Instructions: All questions are compulsory. Figures on the right indicate max. marks. All symbols have their usual meanings.

Q. 1] **Solve any three**

a] State and prove Cauchy's mean value theorem. Give its geometrical significance. [5]

b] Attempt

1. Evaluate $\int_3^7 (7-x)^{\frac{1}{4}}(x-3)^{\frac{1}{4}}dx$ [3]
2. Find the values of: [2]

$$\int_0^{\frac{\pi}{2}} sin^8\theta d\theta$$

and

$$\int_0^{\frac{\pi}{2}} sin^2\theta cos^5\theta d\theta$$

c] Attempt

1. Show that the function $w = ln(2x + 2ct)$ satisfies the wave equation $\frac{\partial^2 w}{\partial t^2} = c^2\frac{\partial^2 w}{\partial x^2}$. [3]
2. Sketch the level surface for $f(x, y, z) = ln(x^2 + y^2 + z^2)$. [2]

Q.2] Attempt all the following.

a] The lengths a, b and c of the edges of a closed rectangular box are changing with time. At the instant in question $a = 1m, b = 2m, c = 3m$, $\frac{da}{dt} = 1m/sec$, $\frac{db}{dt} = 1m/sec$, $\frac{dc}{dt} = -3m/sec$. At what rates are the box's volume V and surface area S changing at that instant? [4]

b] Find the eigen values and the corresponding eigen vectors of the matrix

$$A = \begin{bmatrix} 3 & 5 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{bmatrix}$$

[4]

c] Evaluate $\int_0^\infty e^{-y^3}\sqrt{y}dy$ [2]

OR

State whether Rolle's theorem is applicable to the function $f(x) = x - x^3$. If so, find c.

Q.3] Attempt all the following.

a] By using Lagrange Multipliers method, find the points on the sphere $x^2 + y^2 + z^2 = 4$ farthest from the point $(1, -1, 1)$.

[4]

b] Find all the local maximum, local minimum and saddle points of the function $f(x, y) = 8x^3 + y^3 + 6xy$. [3]

c] Evaluate

$$\int_0^{2a} x^m \sqrt{(2ax - x^2)}\ 'dx$$

[3]

OR

Evaluate using L'H'ôpitals rule:

$$\lim_{x \to 0} \left[\frac{1}{x} - \frac{ln(1+x)}{x^2} \right]$$

Q.4] Attempt **ALL** the following. [9]

a] Find the points where extreme values of the following function may occur.

$$f(x) = \int_0^{\infty} \frac{e^{-y} \cos(xy)}{y}\ dy$$

b] Sketch the region of integration, reverse the order of integration, and evaluate the integral.

$$\int_0^{\pi} \int_x^{\pi} \frac{\sin y}{y}\ dydx$$

c] Evaluate $\int_0^{\sqrt{2}} \int_0^{3y} \int_{x^2+3y^2}^{8-x^2-y^2} dzdxdy.$

Q. 5] Attempt **ANY FOUR** of the following. [16]

a] Evaluate

$$\int_0^{\pi} \int_0^{x} x \sin y\ \ dydx \quad .$$

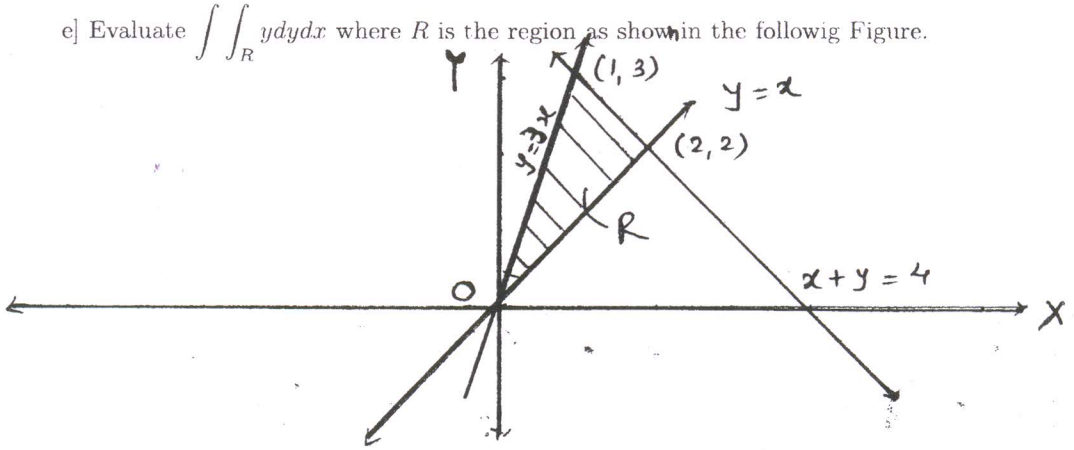
b] Change the cartesian integral to equivalent polar integral and then evaluate it.

$$\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} x^2 + y^2 - 3x\ \ dydx$$

c] A solid ball is bounded by the sphere $\rho = a$. Find the moment of inertia and radius of gyration about the z-axis if the density is $\delta(\rho, \phi, \theta) = r = \rho \sin \phi$.

d] Find the area of the region bounded by the parabola $x = y - y^2$ and line $y = -x$.

e] Evaluate $\int \int_R ydydx$ where R is the region as shown in the followig Figure.



Q. 6] Calculate centroid of the area bounded by the parabola $x^2 + 4y - 16 = 0$ and x-axis. [5]