COLLEGE OF ENGINEERING, PUNE

(An Autonomous Institute of Government of Maharashtra.)
SHIVAJI NAGAR, PUNE - 411 005

END Semester Examination

IS-501-28, MA(ILE)-14001, DE-09023- Linear Algebra

Course Name: B.Tech/M.Tech. Branch Name: All branches

Semester Name: V & VII (UG) / I (PG)

Year: 2014-2015 Max.Marks:60

Duration: 3 Hours Time:- 2 p.m.-5 p.m. **Date:20-Nov-2014**

Instructions: **MIS No.** | | | | | | | | | |

1. Figures to the right indicate the full marks.
2. Mobile phones and programmable calculators are strictly prohibited.
3. Writing anything on question paper is not allowed.
4. Exchange/Sharing of anything like stationery, calculator is not allowed.
5. Assume suitable data if necessary.
6. Write your Seat Number on Question Paper

Q.1 Solve the following system using Doolittle's method (LU decomposition). [5]

$$\begin{aligned} 2x + y + z &= 5 \\ 4x - 6y &= -2 \\ -2x + 7y + 2z &= 9 \end{aligned}$$

Q.2 Solve the following system by Gauss-Seidel method (up to 4 iterations) starting with initial approximation (0,0,0). [6]

$$\begin{aligned} 8x + y - z &= 8 \\ x - 7y + 2z &= -4 \\ 2x + y + 9z &= 12 \end{aligned}$$

Q.3 Find the largest eigenvalue (λ), the corresponding eigenvector and the error ϵ of λ for the following matrix by Power method. Start with $x_0 = [1, 1, 1]^t$ and up to x_4. [6]

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

Q.4 Fit a parabola to the given points (x, y) by the method of least squares. [5]
(-1, 3), (1,1), (2,2), (3,6).

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Q.5 Find a particular solution of the following system of differential equations using method of diagonalization, given that $y_1(0) = 3$ and $y_2(0) = 1$. [6]

$$\begin{aligned} y_1' &= 4y_2 \\ y_2' &= 4y_1 + 2 - 16\,t^2 \end{aligned}$$

Q.6 Find the characteristic and minimal polynomials of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. [4]

Q.7 Suppose the characteristic and minimal polynomials of an operator T are $ch(T) = (\lambda - 3)^4(\lambda - 1)^5$ and $m(T) = (\lambda - 3)^2(\lambda - 1)^3$ respectively. Determine all possible Jordan canonical forms of T. [4]

Q.8 (a) Let V be a vector space over a field K and $T : V \to V$ be a linear operator on V. Let W be the set of all fixed points of T i.e. $W = \{x \in V : T(x) = x\}$. Show that W is a subspace of V. [1]

(b) Let $V = \mathbb{C}$ and $K = \mathbb{R}$. Let $B = \{1, i\}$ be an ordered basis for $\mathbb{C}$ over $\mathbb{R}$. Let $T : \mathbb{C} \to \mathbb{C}$ be a linear operator whose matrix relative to the pair B, B is $A = \begin{bmatrix} 3 & 1 \\ 4 & 3 \end{bmatrix}$. Find a basis for the set of all fixed points of T. [3]

Q.9 Let V be a finite dimensional vector space over a field K and $T : V \to V$ be a linear operator on V. Let $A = [T]_{B \to B}$ be the matrix of T relative to some ordered basis B of V. Then we define the determinant of T as $\det(T) = \det(A)$.

Now, let V be the vector space of all real valued functions defined on $\mathbb{R}$ having an ordered basis $B = \{\sin t, \cos t\}$. Find the determinant of the differential operator $D : V \to V$ defined by $D(f(t)) = \frac{df}{dt}$. [4]

Q.10 Let S^3 denote the vector space of all 3×3 real symmetric matrices. Assume that $< P, Q >= \text{tr}(PQ)$ is an inner product on S^3 (Recall that $\text{tr}(A)$ = trace of a square matrix $A_{n \times n}$ is the sum of its diagonal entries$= a_{11} + \cdots + a_{nn}$).

Let $P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ and $Q = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 0 \end{bmatrix}$.

Find an orthonormal basis for the subspace $\text{Span}(P, Q)$ of S^3. [6]

Q.11 Let V be a finite dimensional vector space and T a linear operator on V. Suppose that $\text{rank}(T^2) = \text{rank}(T)$. Prove that $\text{range}(T) \cap \text{nullspace}(T) = \{0\}$. [4]

Q.12 State and prove rank-nullity theorem for linear transformations.

OR

State and prove Cauchy-Schwartz inequality. [6]

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