# College Of Engineering, Pune

## (An autonomous Institute of Government of Maharashtra)

END-SEM EXAM

**(CT(DE)-14010) Graph Theory and Applications**

**Final Year B.Tech. (Computer Engineering and IT)**

Year: 2013-14 Semester: VII Date: 30/11/2014

Duration: 3 hrs 2 to 5 p.m. Max. Marks: 60

Marks

Q.1 (a) Consider the following notations: 4

(i) Maximum size of independent set : $\alpha(G)$

(ii) Maximum size of matching : $\alpha'(G)$

(iii) Minimum size of vertex cover : $\beta(G)$

(iv) Minimum size of edge cover : $\beta'(G)$

Characterize the simple graphs for which the value of each of the above parameter is 1.

(b) For any simple connected graph G, prove each of the following: 6

(i) $\beta(G) >= \alpha'(G)$

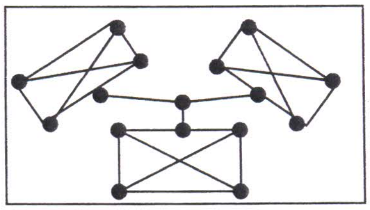
(ii) $\beta'(G) >= \alpha(G)$

(iii) $\beta(G) > = \delta(G)$ ($\delta(G)$ =minimum degree of a vertex in G)

Q.2 (a) If the order of a graph G is even and if S is any set of vertices of the graph then prove that the number of odd components of the graph (G-S) is odd if and only if |S| is odd. 4

(b) Suppose W is the set of universal vertices in a graph G of order n, where n is even. Show that G has a perfect matching if the number of odd components of (G-W) does not exceed |W| and every component of (G-W) is complete. 4

(c) Use Tutte's theorem to show that following graph does not have a perfect matching: 2



Q.3 (a) Prove that the chromatic number of a graph equals the maximum of the chromatic numbers of its components. 4

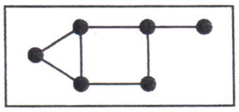
(b) For each of the following graphs, what does Brooks' theorem tell you about the chromatic number of the graph? Find the chromatic number of each graph. 4

(i) The complete graph $K_{20}$ (ii) The bipartite graph $K_{10,20}$

(iii) A cycle with 20 edges (iv) A cycle with 29 edges

(c) Show that $\chi(G) <= 1 + n - \alpha(G)$ ($\chi(G)$: Chromatic number of G) 2

Q.4 (a) Show an independent dominating set and a connected dominating set for the following graph: 4



(b) Suppose a degree sequence $d = (d_1, d_2, ..., d_{2k})$ is defined by $d_{2i} = d_{2i-1} = i$ for $1 <= i <= k$. 6

(i) Draw graphs for k = 2 and k = 3 which have degree sequences as described above.

(ii) Prove that d is graphic for all k without using Havel-Hakimi test.

| | | | Marks |
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| Q.5 | (a) | Prove or disprove: if every vertex of a simple graph G has degree 2, then G is a cycle | 4 |
| | (b) | Show that an edge in a graph G is a bridge if and only if no cycle in G contains that edge. | 4 |
| | (c) | Show that in a graph with n vertices, the length of a path cannot exceed (n-1) and the length of a cycle cannot exceed n. | 2 |
| Q.6 | (a) | Prove that every simple graph with at least two vertices has two vertices of equal degree. | 2 |
| | (b) | Let T be a tree with average degree a. Express the order of T (order of graph: number of vertices in the graph) in terms of a. | 2 |
| | (c) | If both G and its complement are trees, find the order of G. | 2 |
| | (d) | In a tree with 14 terminal vertices, the degree of every non-terminal vertex is either 4 or 5. Find the number of vertices of degree 4 and of degree 5. | 2 |
| | (e) | How many edges are there in a forest with n vertices and k components? Justify your answer. | 2 |

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