

COLLEGE OF ENGINEERING, PUNE

(An Autonomous Institute of Government of Maharashtra.)
SHIVAJI NAGAR, PUNE - 411 005

END Semester Examination

AT-509- Engineering Mathematics for Problem Solving

Course: M.Tech

Branch: Automotive Technology

Semester: Sem I

Year: 2014-2015

Max.Marks:60

Duration: 3 Hours Time: ~~10 am to 4 pm~~ **2 - 5 pm**

Date:20/11/2014

Instructions:

MIS No.

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1. Figures to the right indicate the full marks.
2. Mobile phones and programmable calculators are strictly prohibited.
3. Writing anything on question paper is not allowed.
4. Exchange/Sharing of anything like stationery, calculator is not allowed.
5. Assume suitable data if necessary.
6. Write your MIS Number on Question Paper
7. Answers to Section A and Section B should be in separate answerbooks
8. Exam is Open Book/Open Notes. However no exchange of any material including Calculators, Charts or Tables between students will be allowed

SECTION A (Max Marks : 40)

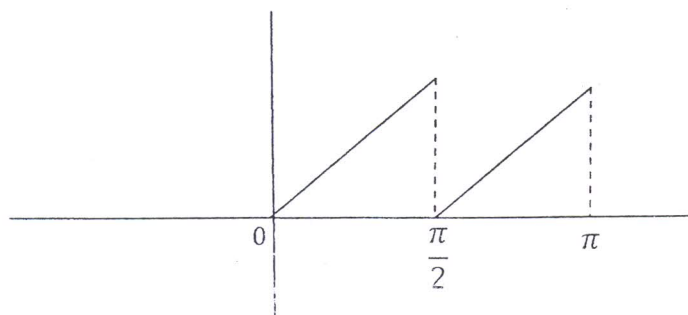
1. Solve the following non-linear equations with the given initial conditions.

(a) $\frac{dy}{dx} = \frac{x^2+x^2y^2}{y+x^3y}; \quad y(0) = 1$ (5)

(b) $\frac{dy}{dx} = \frac{xy^2+4x^3y+x^2}{y^3-x^2y-x^4}; \quad y(0) = 2$ (5)

2. A function is defined on the interval $(0 \leq x \leq \pi)$ as follows.

$$f(x) = \begin{cases} x; & (0 \leq x \leq \frac{\pi}{2}) \\ (x - \frac{\pi}{2}); & (\frac{\pi}{2} < x \leq \pi) \end{cases}$$



- (a) Extend the function to the interval $(-\pi \leq x \leq 0)$ as even and odd extensions and sketch the even and odd extensions. (2)
- (b) Expand the given function into Fourier sine and cosine series. (6)
- (c) State which of these series are uniformly convergent, or not and explain why. (2)

3. The energy of a free vibrating string of length L having tension T_0 and the mass density ρ is given by

$$E = \frac{1}{2} \int_0^L (\rho u_t^2 + T_0 u_x^2) dx$$

where $u(x, t)$ is the solution of the wave equation

$$u_{tt} = c^2 u_{xx}; \quad (0 < x < L); \quad (t > 0); \quad \left(c^2 = \frac{T_0}{\rho}\right) \text{ with boundary conditions}$$

$$u(0, t) = u(L, t) = 0; \quad (t > 0)$$

Express the solution $u(x, t)$ as a Fourier series for the above BC's and apply Parseval's theorem and show that the total energy of the string is constant, i.e.,

$$\frac{dE}{dt} = 0. \quad (10)$$

4. Solve the Poisson equation in the square of side L , with the given BC's.

$$\nabla^2 u = -x; \quad (0 < x < L); \quad (0 < y < L) \text{ with all four sides having zero BC, i.e.,}$$

$$u(x, 0) = u(x, L) = u(0, y) = u(L, y) = 0 \quad (10)$$

PAPER ENDS

SECTION B (Maximum marks 20)

1. Let X be normal with mean 4.2 and variance 0.04. Find c such that
 $P(X \geq c) = 95\%$ $P(X < c) = 10\%$ $P(-c < X - 4.2 \leq c) = 99\%$ [3]
2. Find the variance of $Z = aX + b$ where a and b are constant and X is a Binomial random Variable with variance npq . [3]
3. An airline is planning its staffing needs for the next year. If a new route is approved, it will hire 899 new employees. If a new route is not approved, it will hire only 122 new employees. If the probability that a new route will be approved is 0.37:
 - (i) Using the random variable X for the number of new employees, construct the probability distribution of X .
 - (ii) What is the expected number of new employees hired (to the nearest whole number)? [3]
4. A manufacturer of candy must monitor the temperature at which the candies are baked. Too much variation will cause inconsistency in the taste of the candy. Past records show that the standard deviation of the temperature has been 1.2°F . A random sample of 30 batches of candy is selected, and the sample standard deviation of the temperature is 2.1°F . At the 0.05 level of significance, is there evidence that the population standard deviation has increased above 1.2°F ? [3]
5. The quality control division of a factory that manufactures batteries suspects defects in the production of a model of mobile phone battery which results in a lower life for the product. Until now, the time duration in phone conversation for the battery followed a normal distribution with a mean of 300 minutes and a standard deviation of 30. However, in an inspection of the last batch produced before sending it to market, it was found that the average time spent in conversation was 290 minutes in a sample of 60 batteries. Assuming that the time is still normal with the same standard deviation: Can it be concluded that the quality control suspicions are true at a significance level of 1% ? [4]
6. A feed dealer buys 20% protein feed from a feed manufacturer and resells the feed to local ranchers. The feed dealer is interested in checking to make certain that the feed does not average less than 20% protein. Carry out a hypothesis test of the relevant null hypothesis at the 5% significance level if a sample of 10 feeds showed a mean of 19.87 percent protein content with a standard deviation of 0.19 percent. Show all of your calculations and justify your conclusion. [4]