Total No. of Questions:6 Total No. of pages:2

End Semester Examination

(AS 101) ENGINEERING MATHEMATICS –I

Programme : F.Y. B.Tech

Year : 2005 – 06 Date:

Duration : 3 Hrs. Max.Marks:60

Instruction

1 **Question I is compulsory.**
2 Answer any **Two** sub-questions from question no.II to VI
3 Neat diagrams must be drawn whenever necessary
4 Assume suitable data if necessary
5 Figures to the right indicates full marks
6 Use of any non programmable calculator is allowed
7 Start answer of each question on new page.

Q I Choose the correct option 10

1) The real value of θ for which the expression

$\frac{1+i\cos\theta}{1-2i\cos\theta}$ is real number is

a) $2n\pi \pm \frac{\pi}{2}$ b) $3n\pi \pm \frac{\pi}{2}$ c) $2n\pi \pm \frac{\pi}{4}$ d) None of these above

2) For the interval $a < \xi < b$, Lagrange's mean value theorem satisfies the condition,

a) $f(a) = f(b)$ b) $\frac{f(a)-f(a)}{b-a} = f^1(\xi)$

c) $\frac{f(b)-f(a)}{g(b)-g(a)} = \frac{f^1(\xi)}{g^1(\xi)}$ d) $\frac{f(a)-f(a)}{a-b} = f^1(\xi)$

3) The value of $\lim_{x \to 0}\left[\frac{(1+x)^n - 1}{x}\right]$ is

a) n/2 b) n c) –n/2 d) –n

4) For $u = \sin\left(\frac{x-y}{x+y}\right)^{1/2}$ the value of $\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right)$ is

a) ½ b) 1 c) 0 d) none of these.

5) An nxn homogeneous system of equations AX = B is given.
The rank ρ(A,B) = r , r < n , then system has,
a) Unique solution b) r solution c) Infinite solutions d) No solution

QII (A) If $z = -1 + i\sqrt{3}$ and n is an integer , Prove that $2^{2n} + 2^n z^n + z^{2n}$ is zero if n is not multiple of '3' 05

(B) If $\sin(A+iB) = x+iy$ prove that $\frac{x^2}{\cosh^2 B} + \frac{y^2}{\sinh^2 B} = 1$ & $\frac{x^2}{\sin^2 A} - \frac{y^2}{\cos^2 A} = 1$

(B) Considering the functions $f(x) = 1/x^2$ and $g(x) = 1/x$, prove that 'c' of Cauchy's mean value theorem is the harmonic mean of a and b. **05**

(C) Prove that if $0 < a < b < 1$, $\frac{b-a}{\sqrt{1-a^2}} < (\sin^{-1}b - \sin^{-1}a) < \frac{b-a}{\sqrt{1-b^2}}$ **05**

And deduce that $\frac{\pi}{6} - \frac{1}{2\sqrt{3}} < \sin^{-1}\frac{1}{4} < \frac{\pi}{6} - \frac{1}{\sqrt{15}}$

Q.IV

(A) Expand $\log\tan\left(\frac{\pi}{4} + x\right)$ in ascending powers of x. **05**

(B) Evaluate $\lim_{x \to 0}\left(\log\frac{1}{x}\right)^{\log(1-x)}$ **05**

(C) Solve the following system of equations by Gauss-elimination method **05**

$$2x_1 + x_2 - 5x_3 + x_4 = 8$$
$$x_1 + 3x_2 - 6x_4 = -15$$
$$2x_2 - x_3 + 2x_4 = -5$$
$$x_1 + 4x_2 - 7x_3 + 6x_4 = 0$$

Q.V

(A) If $u = \frac{x^3 + y^3}{y\sqrt{x}} + \frac{1}{x^7}\sin^{-1}\left[\frac{x^2 + y^2}{x^2 + 2xy}\right]$, find the value of **05**

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} + x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$$ at the point (1,2).

(B) If $u^3 + v^3 + w^3 = x + y + z; u^2 + v^2 + w^2 = x^3 + y^3 + z^3; u + v + w = x^2 + y^2 + z^2$

show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = \frac{(x-y)(y-z)(z-x)}{(u-v)(v-w)(w-u)}$ **05**

(C) Find the points on the surface $z^2 = xy + 1$, at a least distance from the origin. **05**

Q.VI

(A) Find values of a and b such that the system of simultaneous equations **05**
$2x - y + 3z = 2$; $x + y + 2z = 2$; $5x - y + az = b$
have (1) No solution. (2) A unique solution. (3) An infinite number of solutions.

(B) Find the eigen values and eigen vectors of the matrix **05**

$$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

(C) Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$ and use it to find A^{-1}. **05**

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