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End Semester Examination

(AS101) ENGINEERING MATHEMATICS-I

Programme: F. Y. B.Tech

Year 2005-06

Duration: 3 hrs

Date:

Max. Marks: 60.

Instructions to candidates:

1. Question No. 1 is **compulsory**.
2. Solve any **two** sub-questions from Q.No. **II to VI.**
3. Figures to the right indicate full marks.
4. Draw neat diagrams wherever necessary.
5. Assume suitable data, if required.
6. Use of only non-programmable calculator is allowed.
7. Start answer of each question on new page.

Q.I Choose the correct option. **10**

i) If $(\alpha+i\beta)=\frac{1}{a+ib}$ then $(\alpha^2+\beta^2).(a^2+b^2)$ is

(a) 1 (b) $\sqrt{2}$ (c) can not be determined (d) None of these.

ii) In the interval (-1,0) the function f(x) = x + (1/x) is

(a) Monotonically increasing (b) monotonically decreasing

(c) Both (d)None of these

iii) The Maclaurin's series expansion of f(x) always contains powers of

(a) x + h (b) x (c) x-1 (d) none of these

iv) The degree of homogeneous function $z=\sqrt{\frac{x^{3/2}+y^{3/2}}{x^2+y^2}}$ is

(a) $\frac{-1}{2}$ (b) $\frac{-1}{\sqrt{2}}$ (c) $\frac{-1}{4}$ (d) $\frac{1}{4}$

v) The nature of eigen vectors of the matrix $\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$ is

(a) Exactly 2 independent vectors (b) Only 1 vector

(c) All non zero vectors (d) none of these

Q.II

(A) If $(1+x)^6+x^6=0$, show that $x=-\frac{1}{2}-\frac{i}{2}\cot\frac{\theta}{2}$ **05**

Where $\theta=\frac{(2n+1)\pi}{6}$, n = 0, 1, 2, 3, 4, 5.

(B) If $u+iv=\frac{1}{i}\log\left(\frac{1+ie^{i\theta}}{1-ie^{i\theta}}\right)$, prove that $u=\frac{\pi}{2}$ and $v=\log(\sec\theta+\tan\theta)$ **05**

(C) The center of a regular hexagon is at the origin and one vertex is given by 1+ i on Argand's diagram. Find the remaining vertices. **05**

Q.III

(A) If $y=e^{a\sin^{-1}x}$, prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+a^2)y_n=0$ **05**

(C) The center of regular hexagon is at the origin and one vertex is given by $\sqrt{3}+i$ on the Argand diagram. Determine other vertices. 05

QIII) (A) Using Cauchy's mean value theorem show that 05

$$\frac{\sin b-\sin a}{\cos a-\cos b}=\cot\xi \quad , \quad o<a<\xi<b<\frac{\pi}{2}, \text{ Hence deduce that } \xi=\pi/4$$

(B) If $y=(\sin^{-1}x)^2$, show that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-n^2y_n=0$ 05

(C) Prove that if a <1, b <1 and a < b then $\frac{b-a}{\sqrt{1-a^2}}<\sin^{-1}b-\sin^{-1}<\frac{b-a}{\sqrt{1-b^2}}$.

Hence show that $\frac{\pi}{6}-\frac{1}{2\sqrt{3}}<\sin^{-1}\frac{1}{4}<\frac{\pi}{6}-\frac{1}{\sqrt{15}}$ 05

Q IV) (A) Prove that $\log(1+e^x)=\log 2+\frac{x}{2}+\frac{x^2}{8}-\frac{x^4}{192}+------$ 05

(B) Evaluate 05

$$\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{1/x^2}$$

(C) Solve the following system of equation by Gauss- Seidal method, 05
20 x + y – 2 z = 17, 3 x +20 y – z = -18, 2 x- 3y + 20 z = 25

QV) (A) If $u=\sin^{-1}\left[\frac{x+2y+3z}{x^8+y^8+z^8}\right]$ find value of $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}$ 05

(B) If u = x + y, uv = y + z , uvw = z 05

Show that $\frac{\partial(x,y,z)}{\partial(u,v,w)}=u^2v$

(C) Show that the approximate change in the angle 'A' of a triangle ABC due to small change $\delta a,\delta b,\delta c$ in the sides a,b,c respectively is given by 05

$$\delta A=\frac{a}{2\Delta}[\delta a-\delta b\cos c-\delta c\cos B]$$, where Δ is the area of triangle ,

Hence verify that $\delta A+\delta B+\delta C=0$

QVI (A) Are the vectors $X_1=(3, 2, 7)$ $X_2=(2, 4, 1)$ $X_3=(1, -2, 6)$ linear dependent ? If so, find relation between them. 05

(B) Test for consistency and solve if consistent, the system of equations 05
$x_1+2x_2+2x_3=1$, $2x_1+2x_2+3x_3=3$, $x_1-x_2+3x_3=5$.

(C) Verify Cayley –Hamilton theorem for matrix 05

$$A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$$

Hence find its inverse.