End Semester Examination
(AS105) ENGINEERING MATHEMATICS-II

Programme: F. Y. B.Tech
Year 2005-06 Date:
Duration: 3 hrs Max. Marks: 60.

Instructions to candidates:
1. All questions are compulsory.
2. **Solve any two sub-questions from Q.No. II to VI.**
3. Figures to the right indicate full marks.
4. Draw neat diagrams wherever necessary.
5. Assume suitable data, if required.
6. Use of only non-programmable calculator is allowed.
7. Start answer of each question on new page.

Q.I Choose the correct option. **10**

i) If $f(x)$ is a periodic function of period p and $a \neq 0$, then $f\left(\frac{x}{a}\right)$

(a) is a periodic function of period p

(b) is a periodic function of period ap

(c) is a periodic function of period $\frac{p}{a}$

(d) may not be a periodic function.

ii) The value of $\iint_R \sin x dxdy$, where R is the region bounded by $x = -\pi$, $x = \pi$, $y = -\pi$ and $y = \pi$ is

(a) 0 (b) 4π

(c) 2π (d) -4π.

iii) The value of $\frac{d}{dt}\operatorname{erf}\left(\sqrt{t}\right)$ is

(a) $\frac{e^{-\sqrt{t}}}{\sqrt{\pi t}}$ (b) $\frac{e^{-t}}{\sqrt{\pi t}}$

(c) $\frac{e^{t}}{\sqrt{\pi t}}$ (d) $\sqrt{\pi t}\; e^{-t}$.

iv) The differential equation of the family of curves represented by the equation

$$y = C_1 e^{2x} + C_2 e^{-2x} + C_3 \cos 2x + C_4 \sin 2x,$$

where C_1, C_2, C_3, C_4 are arbitrary constants is

(a) $\frac{dy}{dx} - 16y = 0$ (b) $\frac{d^2y}{dx^2} - 16y = 0$

(c) $\frac{d^4y}{dx^4} - 16y = 0$ (d) $\frac{d^3y}{dx^3} - 16y = 0.$

v) The area of the region R, where $R = \{(x, y) : -2 \le y \le 2; x \ge 2\}$ is

(a) 4 (b) 8 (c) 0 (d) none of these.

Q.II

(A) Find the Fourier series expansion for periodic function $f(x)$, where

$$f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$

and $f(x+2\pi) = f(x)$. **05**

(B) Obtain Fourier series expansion of $f(x) = \cos ax$ in the interval $(0, 2\pi)$, where a is not an integer. Deduce that

$$\pi \cot 2\pi a = \frac{1}{2a} + a\sum_{n=1}^{\infty} \frac{1}{a^2 - n^2}.$$ **05**

(C) Find the half-range sine series for the following function:

$$f(x) = \begin{cases} \frac{1}{4} - x, & 0 < x < \frac{1}{2} \\ x - \frac{3}{4}, & \frac{1}{2} < x < 1. \end{cases}$$ **05**

Q.III

(A) If $I_n = \int_0^{\pi/4} \frac{\sin(2n-1)x}{\sin x} dx$ then prove that

$n(I_{n+1} - I_n) = \sin \frac{n\pi}{2}$ and hence, find I_3. **05**

(B) Evaluate $\int_0^{\infty} \frac{e^{-x}}{x}\left(a - \frac{1}{x} + \frac{1}{x}e^{-ax}\right)dx.$ **05**

(C) Prove that $\int_1^{\infty} \frac{x^{\frac{n}{2}-1}}{(1+x)^n} dx = \frac{1}{2} B\left(\frac{n}{2}, \frac{n}{2}\right).$ **05**

Q.IV

(A) Trace the cycloid $x = a(t - \sin t)$, $y = a(1 - \cos t)$. **05**

(B) Show that $\int_0^a \int_0^y \dfrac{x\,dx\,dy}{\sqrt{(a^2 - x^2)(y - x)(a - y)}} = \pi a.$ **05**

(C) Evaluate $\iiint z^2\,dx\,dy\,dz$ over the volume common to sphere $x^2 + y^2 + z^2 = a^2$ and cylinder $x^2 + y^2 = ax.$ **05**

Q.V

(A) Solve: $xy \log\left(\dfrac{x}{y}\right)dx + \left(y^2 - x^2 \log\left(\dfrac{x}{y}\right)\right)dy = 0.$ **05**

(B) Solve: $\left(y + \dfrac{y^3}{3} + \dfrac{x^2}{2}\right)dx + \left(\dfrac{x + xy^2}{4}\right)dy = 0.$ **05**

(C) Solve: $\sin y \dfrac{dy}{dx} = \cos x(2\cos y - \sin^2 x)$ **05**

Q.VI

(A) According to Newton's law of cooling, the rate at which a substance cools in moving air is proportional to the difference between the temperature of the substance and that of the air. If the temperature of the air is 30°C and the substance cools from 100°C to 70°C in 15 minutes, find when the temperature will be 40°C. **05**

(B) Find by double integration the area inside the circle $r = a\sin\theta$ and outside the cardioide $r = a(1 - \cos\theta)$. **05**

(C) Find the volume of region bounded by paraboloid $x^2 + y^2 = 2z$ and cylinder $x^2 + y^2 = 4.$ **05**

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