PIET's College of Engineering, Pune

(An Autonomous Institute of Government of Maharashatra)

End Semester Examination

(AS 101) Engineering Mathematics-I

Program: F. Y. B. Tech. **Branch: All branches**

Duration: 3 Hours. **Max. Marks: 60**

Instructions:

(1) **All Questions are Compulsory.**

(2) **Figures to the right indicate full marks.**

Q.I Choose the correct option for the following. 10

(i) If $a = e^{i2\alpha}, b = e^{i2\beta}, c = e^{i2\gamma}, d = e^{i2\delta}$ then the value of $\sqrt{\frac{ab}{cd}} + \sqrt{\frac{cd}{ab}}$ is

(a) $2\cos(\alpha + \beta + \gamma - \delta)$ (b) $2\cos(\alpha + \beta - \gamma - \delta)$

(c) $2\cos(\alpha - \beta + \gamma - \delta)$ (d) $2\cos(\alpha - \beta - \gamma + \delta)$

(ii) If $f'(x) > 0$ for all $x \in (a,b)$ then $f(x)$ is

(a) strictly increasing (b) strictly decreasing

(c) both (d) none of these

(iii) Using Rolle's theorem , the real roots for the equation $x^3 - 6x^2 + 15x + 3 = 0$ are

(a) 1 (b) 2 (c) 3 (d) none of these

(iv)

(iv) Degree of homogeneous function $\left(\frac{x + 2y + 3z}{\sqrt{x^8 + y^8 + z^8}}\right)^{\frac{1}{3}}$ is

(a) 3 (b) $\frac{-1}{3}$ (c) -1 (d) -3

(v) If the Eigen values of matrix A are $1, -2, 3$ then the eigen values of the matrix $3I - 2A + A^2$ are

(a) 2,11,6 (b) 3,11,18 (c) 2,3,6 (d) 6,3,11

Q.II Attempt any Two of the following:

(A) Prove that the n^{th} roots of unity form a geometric progression. Also show that the sum of these **n** roots is zero & their product is $(-1)^{n-1}$. 5

(B) With usual notations , if $i^{\alpha + i\beta} = \alpha + i\beta$ then prove that $\alpha^2 + \beta^2 = e^{-(4n+1)\pi\beta}$. 5

(C) Find the complex number **z** if $|z+i|=|z|$ & $\arg\left(\frac{z+i}{z}\right)=\frac{\pi}{4}$. 5

Q. III Attempt any Two of the following.

(A) If $(\sin^{-1}x)^2 = a_0 + a_1x + a_2x^2 + ----$ then, using the Leibnitz's theorem show that $(n+1)(n+2)\cdot a_{n+2} = n^2 a_n$ 5

(B) Prove that $\sqrt{\frac{1-x}{1+x}} < \frac{\log(1+x)}{\sin^{-1}x} < 1$ using Cauchy's mean value theorem. 5

(C) If $z = f(x,y)$, $x = e^u \cos v$, $y = e^u \sin v$ then show that 5

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = e^{-2u}\left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2}\right)$$

Q. IV Attempt any Two of the following.

(A) Evaluate $\lim_{x\to\infty}\left(\frac{a_1^{\frac{1}{x}} + a_2^{\frac{1}{x}} + --------- + a_n^{\frac{1}{x}}}{n}\right)^{nx}$ 5

(B) If $u^3 + v + w = x + y^2 + z^2$, $u + v^3 + w = x^2 + y + z^2$ 5
$u + v + w^3 = x^2 + y^2 + z$ then prove that :

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = \frac{1 - 4(xy + yz + zx) + 16xyz}{2 - 3(u^2 + v^2 + w^2) + 27u^2v^2w^2}$$

(C) Find the stationary point of $x^2 + y^2 + z^2$ subject to $ax^2 + by^2 + cz^2 = 1$ & $lx + my + nz = 0$ using Lagrange's multiplier method. 5

Q. V Attempt any Two of the following

(A) Test the consistency , if consistent find the solution of the system of equations given: 5

$$5x + 3y + 7z = 4$$
$$3x + 26y + 2z = 9$$
$$7x + 2y + 10z = 5$$

(B) Solve the following system of equations by Gauss Jordan elimination method : 5

$$10x_1 - 7x_2 + 3x_3 + 5x_4 = 6$$
$$-6x_1 + 8x_2 - x_3 - 4x_4 = 5$$
$$3x_1 + x_2 + 4x_3 + 11x_4 = 2$$
$$5x_1 - 9x_2 - 2x_3 + 4x_4 = 7$$

(C) Examine the linear dependence of the following vectors. If linearly dependent find the relation between them : 5

$$X_1 = [3\ 2\ 7]^T \quad , \quad X_2 = [2\ 4\ 1]^T \quad , \quad X_3 = [1\ -2\ 6]^T$$

Q. VI Attempt any Two of the following:

(A) Find the Eigen values & corresponding eigen vectors of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ 5

(B) Find the inverse of given matrix **A** Using Cayley Hamilton theorem $A = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$ 5

(C) Solve the following system of equations by Jacobi's Iterative method upto 4th iteration: 5

$$10x - 2y - 2z = 6$$
$$-x + 10y - 2z = 7$$
$$-x - y + 10z = 8$$