College of Engineering, Pune
Department of Mathematics
F.Y.B.Tech. (All Branches)
MA 102 Engineering Mathematics II
End Semester Examination

Date: April 2009

Max. Time: 3 Hrs.
Max. Marks: 50

Instructions:

(1) All questions are compulsory.
(2) Figures to right indicates full marks.
(3) All symbols have their usual meanings.
(4) Write new question on new page.
(5) Each question carry equal marks.

Que-1) Attempt the following.

(1) The series $\sum_{n=1}^{\infty} \frac{cos n\pi}{n^2+1}$ is absolutely convergent.(**True / False**) [1]
(2) Radius of convergence of the series $1 + \frac{x}{1} + \frac{2!}{2^2}x^2 + \frac{3!}{3^3}x^3 + ...$ is...... [1]
(3) State *Weierstrass's M-Test* for convergence. [1]
(4) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{x^{2(n-1)}}{(n+1)\sqrt{n}}$, where $-1 < x < 1$. [2]
(5) Test the convergence of the series whose n^{th} term is $a_n = sin(\frac{1}{n})$. [2]
(6) Test the convergence of the series
$\sum_{n=1}^{\infty} n^{-log x}$ [3]

OR

Test the convergence of the series $\sum_{n=1}^{\infty} \frac{3n+1}{4n+3} x^n$.

Que-2) Attempt the following

(1) The period of the function $f(x) = sin x + \frac{1}{2}sin 2x + \frac{1}{3}sin 3x + \frac{1}{4}sin 4x$ is..........(*Justify your answer*). [2]
(2) State *Dirichlet conditions.* [2]
(3) Find *Fourier* series of

$$f(x) = \begin{cases} 0 & ; -c < x < 0 \\ 1 & ; 0 < x < c. \end{cases}$$

Find the value of *Fourier* series at point of discontinuity $x = 0$. [2]
(4) Find *Fourier* series of

$$f(x) = \begin{cases} 0 & ; -\pi \le x \le 0 \\ x^2 & ; 0 < x < \pi. \end{cases}$$

which is assumed to be periodic with period 2π. [4]

OR

Show that in the interval (0, 1)

$$cos \pi x = \frac{8}{\pi} \sum_{n=1}^{\infty} \frac{n}{4n^2-1} sin 2n\pi x.$$

Que-3) Attempt the following.

(1) The value of the integral $\int_0^{\frac{\pi}{2}} sin^4 x cos^5 x dx$ is......... [1]

(2) State *Leibnitz's* rule of differentiation under integral sign and hence evaluate $\int_0^1 \frac{x^\alpha - 1}{\log x} dx, \alpha \geq 0$. [2]

(3) Evaluate $\int_0^\infty e^{-x}[\frac{\sin ax}{x}]dx$ [3]

(4) Show that $\int_0^\infty \frac{x^2}{(1+x^4)^3} dx = \frac{5\pi\sqrt{2}}{128}$ [4]

OR

Define *Error function* and prove that

$$\frac{d}{dx}[erf(ax)] = \frac{2a}{\sqrt{\pi}} e^{-a^2x^2}$$

Que-4) Attempt the following.

(1) Change the order of integration $\int_0^2 \int_1^{e^x} f(x, y)dxdy$ [1]

(2) Find the asymptote(s) of the curve $x^3 + y^3 = 3axy$. [2]

(3) Trace the curve $y = x + \frac{1}{x}$. [3]

(4) Changing to the polar coordinates, evaluate

$\int_0^a \int_{\sqrt{ax-x^2}}^{\sqrt{a^2-x^2}} \frac{xy}{x^2+y^2} e^{-(x^2+y^2)} dxdy \; ; \; y > 0$. [4]

OR

Evaluate $\int_0^1 \int_0^x \int_0^{x+y} (x + y + z)dxdy$

Que-5) Attempt the following.

(1) Find the mean value of $e^{-(x^2-y^2)}$ over $x^2 + y^2 = 1$. [2]

(2) Find the mean height of portion of parabola $y = -x^2 + 6x - 6$ which lies above x-axis. [2]

(3) Find the volume bounded by the paraboloid $x^2 + y^2 = az$, the cylinder $x^2 + y^2 = 2ay$ and the plane $z = 0$. [3]

(4) Find the mass of the octant of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, the density at any point (x, y, z) being $kxyz$. [3]

OR

Find the volume of region bounded by paraboloid $x^2 + y^2 = 2z$ and cylinder $x^2 + y^2 = 4$.