College of Engineering, Pune

END SEMESTER EXAM Nov/Dec 2009

F.Y.B.Tech.

MA 101 Engineering Mathematics I

Day and Date: Sunday 6-12-09 — Max.Marks: 50

Timing: 10 am to 1 pm — Duration: 3 hours

Instructions:

1. All questions are compulsory.
2. Assume suitable data, if necessary.
3. All symbols have their usual meanings.
4. Figures to right indicates full marks.
5. Attempt any **two sub-questions from Q.No.II to V.**
6. **Answers to all parts of a question must be answered together or should be properly earmarked. Otherwise they will not be graded.**

Q.I. True or False, Justify. (10)

(a) If $y = x\ sin\ x$, then $y_n = x\ sin\left(x + \frac{n\pi}{2}\right) + n\ cos\left(x + \frac{n\pi}{2}\right)$. (2)

(b) The value of $lim_{x\to 1}\left[\frac{x}{x-1} - \frac{1}{log\ x}\right]$ is 2. (2)

(c) If $x = r\ sin\theta\ cos\phi,\ y = r\ sin\theta\ sin\phi,\ z = r\ cos\theta$, then $\frac{\partial x}{\partial r} \neq \frac{\partial r}{\partial x}$. (2)

(d) Let A be a 3×3 matrix with eigen values $1,\ 2, -3$. Then $det(A^T)$ is zero. (2)

(e) Let A be a 2×2 matrix. Then $adj.\ (adj.\ A) = A$. (2)

Q.II (a) Using Leibnitz's Theorem for x^{2n} prove that

$$1 + \frac{n^2}{1!} + \frac{n^2(n-1)^2}{1^2.2^2} + \frac{n^2(n-1)^2(n-2)^2}{1^2.2^2.3^2} + ... = \frac{(2n)!}{(n!)^2}.$$ (5)

(b) Without solving, show that the equation $x^4 + 2x^3 - 2 = 0$ has one and only one real root between 0 and 1. (5)

(c) Change the Laplacian equation in cartesian co-ordinates $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ into polar co-ordinates. (5)

Q.III (a) Expand $\frac{x}{e^x - 1}$ up to the term in x^4 and hence show that (5)

$$\frac{x}{2}.\frac{e^x + 1}{e^x - 1} = 1 + \frac{1}{12}\ x^2 - \frac{1}{720}\ x^4 + ...$$

(b) Find the circle of curvature of the curve $x + y = ax^2 + by^2 + cx^3$ at the origin. (5)

[P.T.O.]

(c) Reduce the matrix A to its normal form and hence obtain its rank where (5)

$$A = \begin{bmatrix} 2 & -4 & 3 & 1 & 0 \\ 1 & -2 & 1 & -4 & 2 \\ 0 & 1 & -1 & 3 & 1 \\ 4 & -7 & 4 & -4 & 5 \end{bmatrix}.$$

Q.IV (a) (i) Evaluate $\dfrac{\partial(r,\theta)}{\partial(x,y)}$ if $r = \sqrt{x^2+y^2}$, $\theta = tan^{-1}\left(\dfrac{y}{x}\right)$. (2)

(ii) A balloon is in the form of right circular cylinder of radius 1.5 m and length 4 m and is surmounted by hemispherical ends. If the radius is increased by 0.01 m and the length by 0.05 m, find the percentage change in the volume of the balloon. (3)

(b) Verify Euler's Theorem for $f = \dfrac{z}{x+y} + \dfrac{y}{z+x} + \dfrac{x}{y+z}$. (5)

(c) Find the maximum and minimum distances of the point (3,4,12) from the unit sphere with center at origin. (5)

Q.V (a) Test for consistency and solve if consistent, the system of equations $5x + 3y + 7z = 4,\ 3x + 26y + 2z = 9,\ 7x + 2y + 10z = 5.$ (5)

(b) Examine for linear dependence or independence the set of vectors [1 0 2 1], [3 1 2 1], [4 6 2 -4], [-6 0 -3 -4]. If dependent, find the relation between them. (5)

(c) If $X_1 = \dfrac{1}{3}\,[2 \quad 1 \quad 2]^T$ and $X_2 = k\,[3 \quad -4 \quad -5]^T$ where $k = \dfrac{1}{\sqrt{50}}$, construct an orthogonal matrix $A = [X_1 \quad X_2 \quad X_3]$. (5)

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