College of Engineering, Pune
Department of Mathematics
MA 101 Engineering Mathematics I
F Y B TECH(ALL BRANCHES)
End Semester Examination - Autumn Semester

Date:09/10/2010 Max.Marks: 50 Max.Time: 180 minutes.

Instructions:

(1) All questions are compulsory.
(2) Figures to the right indicate maximum marks.
(3) All symbols have their usual meanings.

(1) State true or false and justify [10]

a) If $y = e^{ax}\sin(bx + c)$ then $y_n = r^n e^{ax}\sin(bx + c + n\phi)$ where $r = \sqrt{a^2 + b^2}$, $\phi = \tan^{-1}(\frac{b}{a})$.

b) $\lim \left(\frac{\tan x}{x}\right)^{\frac{1}{x^2}}$ as $x \to 0$ is $e^{\frac{-1}{3}}$.

c) At distance $20m$ from the foot of the tower,the elevation of its top is 60^0.If the possible error in measuring distance and elevation are $1\ cm\ and\ 1\ min$.then the approximate error in calculating height is 4.127 cm.

d) The eigen values of A and A' are same.

e) Determine the nature, index and signature of the quadratic form: $3x_1^2 + 5x_2^2 + 3x_3^2 - 2x_2x_3 + 2x_3x_1 - 2x_1x_2$.

(2) Attempt **any two** of the following: [10]

(a) Expand $e^{a\sin^{-1}x}$, by Maclaurian series upto x^3,also find $y_n(0)$ for even and odd values of n.

(b) State and prove Lagrange's Mean Value Theorem, hence find the point on the curve $y = \log x$ at which the tangent to the curve is parallel to the chord joining the points$(1, 0)$ *and* $(e, 1)$.

(c)Use Taylor's Theorem to prove that

$$\tan^{-1}(x + h) = \tan^{-1}(x) + h\sin z\ \sin z - (h\sin z)^2\frac{\sin 2z}{2} + (h\sin z)^3\frac{\sin 3z}{3} + \ldots$$

where $z = \cot^{-1}x$.

(3) Attempt **any two** of the following: [10]

(a) If $x = e^r\cos\theta$, $y = e^r\sin\theta$. Transform $u_{xx} + u_{yy}$ in terms of r and θ.

(b) If $u = \dfrac{x^3 + y^3}{y\sqrt{x}} + \dfrac{1}{x^7}\sin^{-1}(\dfrac{x^2 + y^2}{x^2 + 2xy})$, find the value of $x^2u_{xx} + 2xyu_{yx} + y^2u_{yy} + xu_x + yu_y$ at $x = 1$, $y = 2$.

(c)If $x = u + e^{-v}\sin u$, $y = v + e^{-v}\cos u$, find u_y, v_x.(Using Jacobians).

(4) Attempt **any two** of the following: [10]

(a) Find the dimensions of a rectangular box of maximum capacity whose surface area is given when
i) box is open at the top ii) box is closed. (Use Lagrange's Method) **PTO**

(b) Reduce the matrix A to its normal form hence find its rank.

$$A = \begin{bmatrix} 5 & 6 & 3 & 2 \\ 3 & 2 & 1 & 2 \\ 2 & -1 & 2 & 5 \\ 1 & 2 & 1 & 0 \\ 1 & 3 & -1 & 3 \end{bmatrix}$$

(c) Examine for Linear independence of the vectors $[2,1,3,2,-1]$, $[4,2,1,-2,3]$, $[0,0,5,6,-5]$, $= [6,3,-1,-6,7]$.

(5) Attempt **any two** of the following: **[10]**

(a) Find the eigen values and corresponding eigen vectors for the matrix.

$$\begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$$

Also find algebric and geometric multiplicity of each eigen value.

(b) Verify Cayley-Hamilton theorem for the matrix A and hence find A^{-1}

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$$

Find A^4, express $B = A^8 - 11A^7 - 4A^6 + A^5 + A^4 - 11A^3 - 3A^2 + 2A + I$ as a quadratic polynomial,find B.

(c) Solve the following equations by Gauss-Seidal method.(upto four iterations)

$83x + 11y - 4z = 95;\ 7x + 52y = 13z = 104;\ 3x + 8y + 29z = 71.$

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