College of Engineering, Pune
(An Autonomus Institute of Government of Maharashtra, Pune-411005)
End-Semester Exam May 2012
(MA 102) Engineering Mathematics II

Programme : F.Y.B.Tech. — Brancehs : All

Academic Year : 2011-12 — Date : 6/5/2012

Duration: 9.00 to 12.000 Hrs. — Max. Marks : 50

Instructions:

(1) All questions are compulsory. (2) All symbols have their usual meanings. (3) Figures to the right indicate maximum marks. (4) Begin a new question on new page and solve all the subquestions together. (5) If you use any notations other than standard notations, describe them specifically so that your answers become self explanatory.

Q.1] Attempt all the following: [10]

(A) Sketch the region bounded by the parabolas $x = y^2$ and $x = 2y - y^2$. Express the area of the region as an iterated double integral and find that area.

(B) Evaluate $\int_0^1 x^5(1-x^3)^{5/2}dx$

(C) Find an integrating factor of the differential equation $(2+y)xdy + 2ydx = 0$. Hence obtain the general solution.

(D) Solve $\frac{\sin^2 y}{x}dx + (\sin 2y \ln(3x) - e^y)dy = 0.$

(E) Three solutions of a certain second order nonhomogeneous linear differential equation are

$y_1(x) = \cos 2x + x\sin 2x \qquad y_2(x) = (x-1)\sin 2x \qquad y_3(x) = 2x\sin x\cos x.$

Examining the solutions carefully, write with justification the general solution of this differential equation in the form of $y_h(x) + y_p(x)$.

Q.2]

(A) Find the first moment about $X-$ axis of a thin plate of density $\delta(x,y) = 1$ covering the region under the curve $y = e^{-x^2/2}$ in the first quadrant. [2]

(B) Evaluate $\int\int_R x \ dA$ where R is the region bounded by the lines $y = x$, $y = 3x$ and $x + y = 4$. [3]

(C) Find the polar moment of inertia about the origin of a thin plate covering the region that lies inside the cardioid $r = 1 - \cos\theta$ and outside the circle $r = 1$ if the plate's density function is $\delta(x,y) = \frac{1}{x^2+y^2}$ [3]

OR

(C) State Rule II of Differentiation under integral sign and verify it for $\int_a^{a^2} \frac{dx}{x+a}$. [3]

Q.3]

(A) Solve $\int_0^4\int_0^1\int_{2y}^2 \frac{4\cos(x^2)}{2\sqrt{z}}dxdydz.$ [2]

(B) Attempt any two:

1. With the help of diagram, explain the relation between cartesian and spherical co-ordinates. Using spherical co-ordinates, find the mass of the solid in the form of the positive octant of the sphere $x^2 + y^2 + z^2 = 9$ given that the density at any point is $2xyz$. [6]
2. A solid right circular cylinder is bounded by the cylinder $r = a$ and the planes $z = 0$ and $z = 2$. Find the center of mass if the density is $\delta(r, \theta, z) = z + 1$.
3. Find the volume of the solid right cylinder whose base is the region in xy-plane that lies between the circles $x^2 - x + y^2 = 0$ and $x^2 - 2x + y^2 = 0$ and whose top lies in the plane $z = 3 - y$.

Q.4]

(A) Obtain the general solution of a linear differential equation of first order $y' + p(x)y = r(x)$ where $p(x)$ and $r(x)$ are any given continuous functions of x.

(B) A generator having emf $20\cos 5t$ volts is connected in series with a 10 ohm resistor and an inductor of 2 henrys. If the switch is closed at a time $t = 0$, determine the current at time $t > 0$. Also obtain the steady state solution. [2]

(C) Obtain the particular solution of the differential equation $(y + 3x^4\cos^2(y/x))dx - xdy = 0$ if $y(1) = 0$. [3]

[3]

OR

(C) Find the orthogonal trajectories of the circles with centers on Y-axis and passing through origin. [3]

Q.5]

(A) Find the steady state and transient state motion of the mass-spring system with mass $4kg$, damping $c = 8kg/sec$, spring constant $3kg/sec^2$, and driving force $425\sin 2t$ Newton, by solving the equation $my'' + cy' + ky = r(t)$. When $y(0) = -16$ and $y'(0) = -26$, determine whether the motion is in steady state or not.

(B) Attempt any **Two** : [2]

[6]

1. Obtain the general solution of $(D^3 - D^2)y = e^{2t} + t$.
2. Obtain the general solution of $(x^2D^2 - 2xD + 2)y = x^3\cos x$
3. $y_1(x) = x^2$ is a solution of the homogeneous differential equation $(x^3 - x^2)y'' - (x^3 + 2x^2 - 2x)y' + (2x^2 + 2x - 2)y = 0$. Obtain a second linearly independent solution $y_2(x)$ of the same differential equation.

Q.6]

(A) A differential equation of the form

$$y' = p(x) + q(x)y + r(x)y^2$$

is called Ricati's equation. If a particular solution $y = y_1(x)$ is known, then the general solution is given by $y(x) = y_1(x) + u(x)$, where u satisfies the Bernoulli's equation

$$\frac{du}{dx} - (q + 2ry_1)u = ru^2.$$

Solve the differential equation $y' = x^3(y - x)^2 + y/x$ by identifying it as Ricati's equation. Given that $y_1 = x$ is a particular solution.

(B) Attempt any **One**: [3]

[5]

1. Solve the nonhomogeneous system of linear equations: $y_1' = -3y_1 - 4y_2 + 4e^{-t}$; $y_2' = 5y_1 + 6y_2 - 7e^{-t}$ with $y_1(0) = -1$ and $y_2(0) = 3$.
2. Obtain the general solution of $(D^3 - 2D^2 - D + 2)y = xe^x$ by method of variation of parameters.