Maths

College of Engineering, Pune
Department of Mathematics
MA 101 Engineering Mathematics I (For All Branches)

End semester Examination

Date:26/11/2012 Max. Marks 50 Max. Time 180 minutes

Instructions: All questions are compulsory. Figures on the right indicate maximum marks.

1. (a) Find the local extreme values of the function $f(x) = \frac{4}{x^2-4}$. **[2]**

   (b) A rectangle is inscribed in a semicircle of radius 2. Find the dimensions of the rectangle so that its area is maximum. **[2]**

   (OR)

   Find the point on the parabola $y = x^2$ which is closest to the point $(6, 3)$. **[2]**

   (c) Prove that

$$\lim_{x\to 0} \frac{x\,sin(sinx) - sin^2 x}{x^6} = \frac{1}{18}$$

   . **[4]**

   (OR)

   Prove that

$$\lim_{x\to 0} \left[2\left(\frac{cosh\,x - 1}{x^2}\right)\right]^{1/x^2} = e^{1/12}$$

   . **[4]**

   (d) Let $y^{1/m} - y^{-1/m} = 2x$. By Leibnitz theorem, prove that

   $y = 1 + mx + m^2\frac{x^2}{2!} + m(m^2 - 1^2)\frac{x^3}{3!} + m^2(m^2 - 2^2)\frac{x^4}{4!} + \cdots$. **[5]**

2. (a) Attempt (ANY FIVE) **[10]**

   i. Test for convergence the series $\sum_{n=1}^{\infty} a_n$ if $a_1 = 2, a_{n+1} = \frac{1+sin\,n}{n} a_n, n \geq 1$. If convergent, what conclusion can you draw about the convergence of the sequence $\{a_n\}_{n=1}^{\infty}$? Give reasons for your answer.

   ii. Check whether the series $\sum_{n=1}^{\infty} \frac{5^n - 13^n}{65^n}$ converges. Give reasons for your answer. If it converges, find its sum.

   iii. Given that $F(t) = e^{sint}, g(x, y) = cos(x^2 + y^2)$ and $H(x, y) = F(g(x, y))$, evaluate $\frac{\partial H}{\partial x}$ at the origin using appropriate chain rule.

   iv. Discuss local extreme values of the function $f(x, y) = xy$. Sketch the level curves.

v. Check whether limit of the function $f(x,y) = \frac{x^2 y}{x^2+y^2}$, if $(x,y) \neq (0,0)$ exists at the origin. If yes, define $f(0,0)$ in a way that extends it to be continuous at the origin.

vi. If $w = f(u) + g(v)$, where f and g are differentiable functions, $u = x + iy$ and $v = x - iy$, show that w satisfies Laplace's equation $w_{xx} + w_{yy} = 0$.

(b) The lengths a, b and c of the edges of a closed rectangular box are changing with time. At the instant in question $a = 1m, b = 2m, c = 3m$, $\frac{da}{dt} = 1m/sec$, $\frac{db}{dt} = 1m/sec$, $\frac{dc}{dt} = -3m/sec$. At what rates are the box's volume V and surface area S changing at that instant? Are the box's interior diagonals increasing or decreasing in length? **[3]**

3. (a) Evaluate (ANY ONE) **[2]**

i.

$$\lim_{x\to a}\left(2 - \frac{x}{a}\right)^{tan\left(\frac{\pi x}{2a}\right)}$$

ii.

$$\lim_{n\to\infty}\left[\frac{e^{1/n} + e^{2/n} + \cdots + e^{n/n}}{n}\right]$$

(b) Discuss the solution of the following system completely **[4]**

$$\begin{aligned} 2x_1 + x_2 &= a \\ x_1 + \lambda x_2 - x_3 &= b \\ x_2 + 2x_3 &= c \end{aligned}$$

(c) Solve the following system by Gauss-Elimination method or indicate the non-existance of solution: **[2]**

$$\begin{aligned} 2w + 3x + y - 11z &= 1 \\ 5w + 2x + 5y - 4z &= 5 \\ w - x + 3y - 3z &= 3 \end{aligned}$$

(OR)

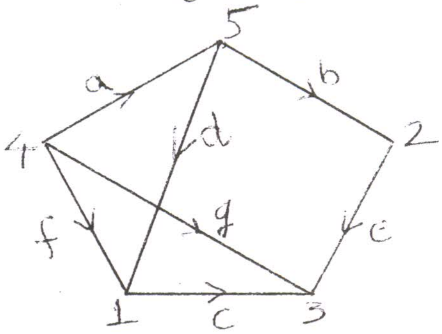
Examine the given vectors for linear independance or dependance: **[2]**
$K_1 = [2, 1, 3]$, $K_2 = [4, 2, 1]$, $K_3 = [0, 0, 5]$, $K_4 = [6, 3, -1]$

(d) The following figure shows a network with nodes (represented by numbers) and branches (represented by alphabets) with one node grounded. The matrix of any such network is defined as $A = [a_{ij}]$ where a_{ij}'s are defined as follows:

$$a_{ij} = \begin{cases} +1 & \text{if branch j leaves node i,} \\ -1 & \text{if branch j enters node i.} \\ 0 & \text{otherwise} \end{cases}$$

i. What is the order of A of the following network?

ii. Write the matrix A of the following network. **[4]**



4. (a) i. Assume that f is differentiable on $a \leq x \leq b$ and that $f(a) < f(b)$. Show that f' is negative at some point between a and b. **[2]**

ii. Suppose that f is differentiable at every point x and that $f(1) = 1, f' < 0$ on $(-\infty, 1)$ and $f' > 0$ on $(1, \infty)$. Show that $f(x) \geq 1, \forall x$. **[2]**

(b) i. If $A = \begin{bmatrix} 3 & 0 & 0 \\ 5 & 7 & 0 \\ 2 & 6 & 1 \end{bmatrix}$, find eigen values of A^3 and A^T. **[1]**

ii. Find eigenvalues and corresponding eigen vectors of $A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$ **[3]**

(OR)

Show that any 5×5 matrix with real entries will have at least one real eigenvalue. Do there exist 2×2 matrices with no real eigenvalues? Justify your answer. **[3]**

(c) i. Justify whether true or false. **[2]**

A. If λ is an eigenvalue of matrix $A_{n \times n}$ and μ is an eigen value of matrix $B_{n \times n}$, then $\lambda + \mu$ is an eigen value of matrix $A + B$.

B. If X is an eigenvector of matrix A corresponding to an eigenvalue λ, then X is also an eigenvector of matrix $A + I$ corresponding to the eigen value $\lambda + 1$.

ii. Do there exist skew-symmetric 3×3 matrices that are orthogonal? Justify your answer. **[1]**

(OR)

Attempt the following: **[1]**

A. Find eigenvalues of 2×2 matrix with trace 9 and determinant 20.

B. If eigenvalues of 3×3 matrix A are 0, 1 and 2, then find the determinant of AA^T.

iii. Find the matrices which describe **[1]**

A. orthogonal projection of R^2 onto the y-axis

B. reflection about the x-axis in R^2.

◇ ◇ ◇ ◇ ◇