College of Engineering, Pune
Department of Mathematics
End Semester Examination Nov/Dec 2013

MA 101- Engineering Mathematics I (F.Y. B.Tech. All Branches)

Date: Max. Marks 60 Max. Time 3 Hours

Instructions:

1. All questions are compulsory.
2. Begin a new Question on new page and answer ALL the subparts of a Question together.
3. You should not begin a sub question on new page.
4. Figures on the right indicate maximum marks.
5. If you are using any symbols that are not standard or not used in the text book then define them properly and use.

Question 1

1. Answer the following :

   (a) Prove that the eigen value of $adj(A)$ is $\frac{|A|}{\lambda}$ where λ is the eigen value of an invertible matrix A. **[2]**

   (b) Illustrate with an example that $rank(A) = rank(B)$ does not imply $rank(A^2) = rank(B^2)$. **[1]**

   (c) Say True or False. Justify your answer. **[1]**
   The set $\{(0, 0)\}$ is linearly independent in $\mathbb{R}^2$.

2. Discuss the solutions of the following system of linear equations completely for all values of λ : **[4]**

$$(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0$$

$$(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z = 0$$

$$2x + (3\lambda + 1)y + 3(\lambda - 1)z = 0$$

   For non-trivial solutions, Find the ratios $x : y : z$ when λ has the largest of these values.

3. (a) Find the principal directions (eigen vectors) and corresponding factors of extension or contraction (eigen values) of the elastic deformation $Y = AX$ with **[3]**

$$A = \begin{bmatrix} 4 & \sqrt{8} \\ \sqrt{8} & 6 \end{bmatrix}$$

   (b) Find the matrix A in the indicated linear transformation $Y = AX$ (any one): **[1]**

   i. Counterclockwise rotation of the cartesian xy-coordinate system in the plane about the origin through an angle $\frac{\pi}{3}$.

   ii. Orthogonal projection of $\mathbb{R}^2$ onto the y-axis.

Question 2

1. How close does the curve $y = \sqrt{x}$ come to the point $(3/2, 0)$? Also find the point(s) on the curve which is closest to this point. [3]

2. Using MVT prove that if $f'(x) = 0$ for all x in an open interval (a, b) then $f(x) = c$ for all $x \in (a, b)$ where c is some constant. [2]

3. If $f(x) = \sqrt[3]{x} + \sqrt[3]{x-1}$ then find [5]

   (a) the domain and range of f

   (b) all the critical points of f and the intervals in which the function is increasing or decreasing.

   (c) the intervals in which the function is concave up or concave down

   (d) sketch the graph of f

4. Show by examples that ratio test fails when $\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = 1$ [2]

Question 3

1. Let u and v be functions of x and y such that $u_x = v_y$ and $u_y = -v_x$. If $x = r\cos\theta$ and $y = r\sin\theta$ then prove that $ru_r = v_\theta$ and $u_\theta = -rv_r$ [4]

2. Divide a number N in three parts such that their product is maximum. Should there be any condition on N? Justify. [4]

3. Prove: If $f(x, y)$ has a local extremum at an interior point (a, b) of its domain and if the first partial derivatives exist there then $f_x(a, b) = f_y(a, b) = 0$. Is the converse true? Justify. [4]

Question 4

1. Find the value of $\frac{\partial x}{\partial z}$ at the point $(1, -1, -3)$ if the equation $xz + y\ln x - x^2 + 4 = 0$ defines x as a function of y and z. [2]

2. Evaluate $\frac{\partial z}{\partial u}$ when $u = 1$ and $v = -2$ if $z = \ln q$ and $q = \sqrt{v+3}\tan^{-1} u$. [2]

3. Explain the difference between level curves and contour curves by an example. [2]

4. Show that the function $w(x, t) = ln\sqrt{x^2 + t^2}$ satisfies the Laplace equation $w_{xx} + w_{tt} = 0$ [2]

5. If $u = f(x, y)$ where $x = e^s \cos t$ and $y = e^s \sin t$. Show that [4]

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = e^{-2s}\left[\frac{\partial^2 u}{\partial s^2} + \frac{\partial^2 u}{\partial t^2}\right]$$

OR

The temperature at a point (x, y) on the circle $x = \cos t, y = \sin t, 0 \leq t \leq 2\pi$ is $T(x, y)$, measured in degrees Celsius. Suppose also that $\frac{\partial T}{\partial x} = 8x - 4y$, $\frac{\partial T}{\partial y} = 8y - 4x$. (i) Find the hottest and coldest points on the circle? (ii) If $T(x, y) = 4x^2 - 4xy + 4y^2$, find the maximum and minimum temperature. [4]

Question 5

1. Fill in the blanks : **[4]**

(i) If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n = 0$. Which of the following statements are logically correct?

[a] $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges since $\lim_{n\to\infty} \frac{1}{n^2} = 0$

[b] $\sum_{n=1}^{\infty} \frac{n}{3n+1}$ diverges since $\lim_{n\to\infty} \frac{n}{3n+1} = (1/3) \neq 0$

[c] $\lim_{n\to\infty} \frac{3n^2}{n+4} \neq 0$ since $\sum_{n=1}^{\infty} \frac{3n^2}{n+4}$ diverges.

Answer: ——————

(ii) $\lim_{n\to\infty} \sqrt[n]{n} =$ ——.

(iii) To decide the convergence of the series $\sum_{n=1}^{\infty} \frac{(\ln n)^n}{n^n}$, —— test is applicable.

(iv) It is given that the power series $\sum_{n=1}^{\infty} (x+5)^n$ converges if $x = -4.5$. Then the series $\sum_{n=1}^{\infty} (x+5)^n$ converges if $x = -4.8$ because ——————.

2. Attempt any **TWO**. **[4]**

(i) Find the radius and interval of convergence of the following power series $\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n}$.

(ii) State Taylor's formula for expansion of a function $f(x)$ at $x = a$. Find the Taylor series generated by $f(x) = x^4 + x^2 + 1$ at $x = -2$.

(iii) By using suitable Maclaurin's series expansions, evaluate $\lim_{z\to 0} \left(\frac{1 - \cos^2 z}{\ln(1-z) + \sin z} \right)$.

3. Prove that the Maclaurin's series for an odd function $f(x)$ defined in an interval $(-c, c)$ contains only odd powers of x. Assume that the series converges to $f(x)$ for all $x \in (-c, c)$. **[2]**

4. Give an example of a function $f(x)$ such that the Taylor's polynomial of order 4 generated by f at $x = a$ is of degree 2. **[2]**