COLLEGE OF ENGINEERING, PUNE

(An Autonomous Institute of Government of Maharashtra.)
SHIVAJI NAGAR, PUNE - 411 005

END Semester Examination

(MA-101) Engineering Mathematics-I

Course: B.Tech Branch: Applied Science

Semester: Sem I

Year: 2014-2015 Max.Marks:60

Duration: 3 Hours Time:- 10.00 am to 1.00 pm **Date:21 November 2014**

Instructions: **MIS No.** | | | | | | | | | |

1. Figures to the right indicate the full marks.
2. Mobile phones and programmable calculators are strictly prohibited.
3. Writing anything on question paper is not allowed.
4. Exchange/Sharing of anything like stationery, calculator is not allowed.
5. Assume suitable data if necessary.
6. Write your MIS Number on Question Paper

Question [I]

(1) Attempt (**any three**) [9]

(a) Solve the following system of linear equations:
$2x_1 - x_2 + 3x_3 = 0$; $3x_1 + 2x_2 + x_3 = 0$; $x_1 - 4x_2 + 5x_3 = 0$.

(b) Find all values of c for which the system of equations $5x + 3y + 2z = 12$; $2x + 4y + 5z = 2$; $39x + 43y + 45z = c$ is consistent. For these values of c, solve the system.

(c) Prove that eigen vectors of a real symmetric matrix corresponding to distinct eigen values are orthogonal to each other.

(d) If the equation $F(x, y, z) = 0$ determines z as a differentiable function of x and y then at points where $F_z \neq 0$, show that $\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}$.
Hence for $F(x, y, z) = xe^y + ye^z + z\ln x - 2 - 3\ln 2 = 0$, find $\frac{\partial z}{\partial x}$ at $(1, \ln 2, \ln 3)$.

(2) Describe the matrix A corresponding to the orthogonal projection of $\mathbb{R}^3$ onto the plane $y = x$ with details. Determine the principal values and corresponding principal directions of this linear transformation either algebraically or geometrically. [3]

(3) Define the null space of a linear transformation from one vector space to another. Prove that the null space of a linear transformation has dimension 0 if and only if the transformation is one-to-one. [3]

Question [II]

(1) Attempt (**any three**) [6]

(a) If $w = tan^{-1}(\frac{y}{x})$, verify that $w_{xy} = w_{yx}$.

(b) Determine the absolute extreme values of $g(x) = \sqrt{5 - x^2}$.

(c) Assume that f is continuous on $[a, b]$ and differentiable on (a, b). Also assume that $f(a)$ and $f(b)$ have opposite signs and that $f' \neq 0$ between a and b. Show that $f(x)$ is zero exactly once between a and b.

(d) For $0 < a < b < 1$, prove that $\frac{b-a}{\sqrt{1-a^2}} < sin^{-1}b - sin^{-1}a < \frac{b-a}{\sqrt{1-b^2}}$.

(2) Fill in the blanks:
If f is an even function defined over $\mathbb{R}$ then in any open interval containing 0, f is _____(increasing / decreasing / neither increasing nor decreasing). If it has a local maximum value M at $x = c$ then $f(-c) =$ _____ and f has local _____(maximum / minimum) at $x = -c$ because _____ . [3]

(3) A 1125 ft^3 open top rectangular tank with a square base x ft on a side and y ft deep is to be built with its top flush with the ground to catch runoff water. The costs associated with the tank involve not only the material from which the tank is made but also an excavation charge proportional to the product xy. If the total cost is $c = 5(x^2 + 4xy) + 10xy$, what values of x and y will minimize it? [3]

(4) From Jensen's inequality, derive

(a) the A.M.-G.M. inequality

(b) any one trignometric characteristion of an equilateral triangle. [3]

Question [III]

(1) If $z = f(x, y)$, define the partial derivative of z with respect to x at the point (x_0, y_0). Explain its geometrical meaning. [2]

(2) The derivative of $f(x, y)$ at P_0 in the direction of $\hat{i} + \hat{j}$ is $2\sqrt{2}$ and in the direction of $-2\hat{j}$ is -3. What is the derivative of f at P_0 in the direction of $-\hat{i} - 2\hat{j}$? [2]

(3) Sketch the curve $x^2 + y^2 = 4$ together with ∇f and the tangent line at the point $(\sqrt{2}, \sqrt{2})$. Also write the equation for this tangent line. [2]

(4) Find the maximum and minimum values of $f(x, y, z) = x - 2y + 5z$ on the sphere $x^2 + y^2 + z^2 = 30$. [2]

(5) Find the absolute maximum and minimum values of $f(x, y) = 2 + 2x + 2y - x^2 - y^2$ on the triangular region in the first quadrant bounded by the lines $x = 0$, $y = 0$ and $y = 9 - x$. [4]

OR

Evaluate $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ at the point $(u, v) = (-2, 0)$, where $w = \ln(x^2 + y^2 + z^2)$, $x = ue^v sinu$, $y = ue^v cosu$, $z = ue^v$.

(6) State (without proof) a criterion to determine the positive definiteness of an $n \times n$ symmetric matrix in terms of certain determinants associated with that matrix. Using this criterion, prove that if an $n \times n$ symmetric matrix A is positive definite, then so is every symmetric $n \times n$ matrix B whose entries are sufficiently close to the respective entries of A. [3]

Question [IV]

(1) Attempt (**any four**) [12]

(a) Test the convergence of the following series:

(i) $\sum_{n=1}^{\infty} \frac{e^{n\pi}}{\pi^{ne}}$, (ii) $\sum_{n=1}^{\infty} \frac{1.3....(2n-1)}{[2.4....(2n)](3^n+1)}$.

(b) Find the radius and the interval of convergence of $\sum_{n=0}^{\infty} \frac{nx^n}{4^n(n^2+1)}$. Show all the details.

(c) Find the Maclaurin's series for $f(x) = cos^2 x$. Hence calculate $cos^2(0.1)$ with an error of less than 10^{-6}.

(d) You drop a ball from t meters above a flat surface. Each time the ball hits the surface after falling a distance h, it rebounds a distance rh, where r is a positive real number but less than 1. Find the total distance the ball travels up and down.

(e) State whether true or false. Justify your answer.

(i) If both $\sum a_n$ and $\sum b_n$ are divergent then $\sum (a_n + b_n)$ is divergent.

(ii) If $\sum a_n$ is convergent then $\{a_n\}$ is convergent.

(2) Write a short note on completeness of the real number system, some of its implications and its importance, with an illustration of what can go wrong without it. [3]

OR

If $\{a_n\}$ and $\{b_n\}$ are two sequences of real numbers such that $\{a_n\}$ converges to m and $\{b_n\}$ converges to t, then prove that $\{a_n + b_n\}$ converges to $m + t$.

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