College of Engineering, Pune
(An Autonomus Institute of Government of Maharashtra, Pune-411005)
End-Semester Exam May 2012
Engineering Mathematics IV

Programme : S.Y.B.Tech.
Academic Year : 2011-12
Duration: 9.00 to 12.000 Hrs.

Brancehs : Comp. and IT
Date : 5/5/2012
Max. Marks : 25+25

Instructions: All questions are compulsory. Figures on the right indicate max. marks. All symbols have their usual meanings. Use seperate answersheets for section I and II.

Section I - Statistics and Probability

Q.1] Attempt any **Three** [9]

A] A random sample of 64 bags of popcorn weighed, on average, 5.40 ounces with a standard deviation of 0.24 ounces. Test the hypothesis that $\mu = 5.5$ ounces against the alternative hypothesis $\mu \neq 5.5$ ounces at the 0.05 level of significance.

B] A service facility opeartes with two service lines. On a randomly selected day, let X be the proportion of time that the first line is in use whereas Y is the proportion of time that the second line is in use. Suppose that the joint probability density function for (X, Y) is given by

$$f(x,y) = \frac{3}{2}(x^2+y^2), \quad 0 \leq x, y \leq 1,$$
$$= 0, \text{ elsewhere.}$$

Determine whether or not X and Y are independent.

C] Past experience indicates that the time required for high school seniors to complete a standardised test is a normal random variable with a standard deviation of 6 minutes. Test the hypothesis that $\sigma = 6$ against the alternative that $\sigma < 6$ if a random sample of 20 highschool seniors has a standard deviation $s = 4.51$ Use a 0.05 level of significance.

D] What sample size should be needed to produce a 95% confidence interval for the mean of the normal disrtibution with $\sigma^2 = 9$ of length 0.4.

Q.2] Attempt any **FOUR** [16]

A] (i)A distributor sells rubber bands in packages of 100 and gurantees that at most 10% are defective. A consumer controls each package by drawing 10 bands without replacement. If the sample contains no defective rubber bands, he accepts the package. Otherwise he rejects it. Find the probability that a customer rejects a package that contains 8 defective bands. [2]

(ii) In a certain manufacturing process it is known that, on the average, 1 in every 100 items is defective. What is the probability that the fifth item inspected is the first defective item found? [2]

B] Compute and interpret the correlation coefficient for the following grades of 6 students selected at random:

Mathematics Grade	70	92	80	74	65	83
English Grade	74	84	63	87	78	90

Test the null hypothesis that $\rho = 0.235$ against the alternative that $\rho > 0.235$. Use a 0.05 level of significance.

C] The following is a portion of a classic data set called the "pilot plot data" in fitting equations to data by Daniel and Wood, published in 1971. The response y is the acid content of material produced by titration whereas the regressor x is the organic acid content produced by extraction and weighing.

y	76	62	66	58	88	70	37	82	88	43
x	123	55	100	75	159	109	48	138	164	28

(a) Fit a simple linear regression, estimate a slope and intercept.

(b) Find the acid content of material produced by titration if organic acid content is given as 90.

D] (i) Show that S^2 is an unbiased estimation of the parameter σ^2. [2]

(ii) Define null hypothesis, Type I error and Type II error. Comment on relation between sample size and possibilities of committing both types of errors. [2]

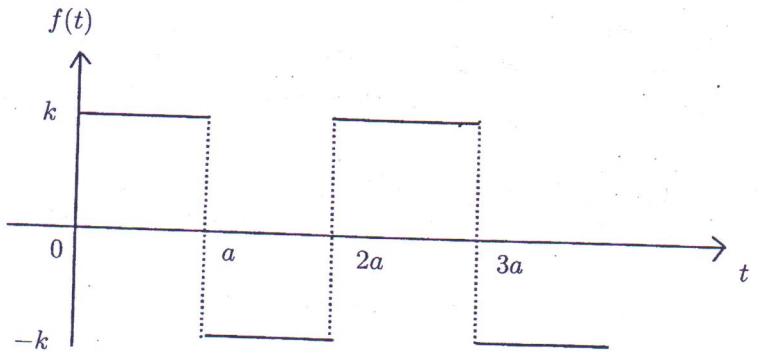
E] If X and Y are random variables with joint probability disribution $f(x, y)$, then prove that

$$\sigma^2_{aX+bY} = a^2\sigma^2_X + b^2\sigma^2_Y + 2ab\sigma_{XY}.$$

Hence find the variance of random variable $Z = -2X + 4Y - 3$ where X and Y are independent random variables with variences $\sigma^2_X = 5$ and $\sigma^2_Y = 3$.

SECTION II

1. Evaluate: $L\{t^2 \sin at\}$.
   Using the theorem on transforms of derivatives. [2]
2. Find $L^{-1}\{\tan^{-1}(\frac{2}{s^2})\}$. [2]
3. State and prove the relation between $\lim f(t)$ as $t \longrightarrow \infty$ and $\lim F(s)$ as $s \longrightarrow 0$. Where $F(s) = L\{f(t)\}$. [2]
4. Find finite Fourier sine and cosine transform of x^2 in $0 < x < L$. [2]
5. Find Fourier transform of $e^{-a^2x^2}$.
   Hence find Fourier cosine transform of e^{-x^2} . [2]
6. Find Laplace's transform of $f(t)$ as shown in figure.



(given $f(t + 2a) = f(t)$) [2]

7. Solve $y''' + 4y'' + 5y' + 2y = 10\ cost$.
   with $y(0) = y'(0) = 0,\ y''(0) = 3$. (Use Laplace's transform). [4]

8. Find Fourier sine transform and Fourier cosine transform of x^{n-1}. Hence deduce that $\frac{1}{\sqrt{x}}$ is self reciprocal under both transforms. Also find $F\{\frac{1}{\sqrt{|x|}}\}$. [4]

9. Solve Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, $y \geq 0$. Subject to the boundary conditions :
$u(x, 0) = f(x), \quad -\infty < x < \infty$
$u(x, y) \longrightarrow 0$, as $y \longrightarrow \infty$. [5]

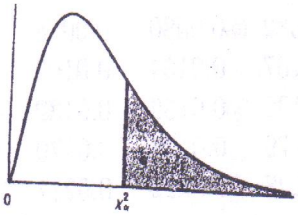
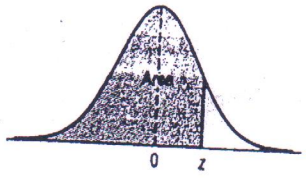


Table A.5 Critical Values of the Chi-Squared Distribution

ν	α									
	0.995	**0.99**	**0.98**	**0.975**	**0.95**	**0.90**	**0.80**	**0.75**	**0.70**	**0.50**
1	0.0^4393	0.0^3157	0.0^3628	0.0^3982	0.00393	0.0158	0.0642	0.102	0.148	0.455
2	0.0100	0.0201	0.0404	0.0506	0.103	0.211	0.446	0.575	0.713	1.386
3	0.0717	0.115	0.185	0.216	0.352	0.584	1.005	1.213	1.424	2.366
4	0.207	0.297	0.429	0.484	0.711	1.064	1.649	1.923	2.195	3.357
5	0.412	0.554	0.752	0.831	1.145	1.610	2.343	2.675	3.000	4.351
6	0.676	0.872	1.134	1.237	1.635	2.204	3.070	3.455	3.828	5.348
7	0.989	1.239	1.564	1.690	2.167	2.833	3.822	4.255	4.671	6.346
8	1.344	1.647	2.032	2.180	2.733	3.490	4.594	5.071	5.527	7.344
9	1.735	2.088	2.532	2.700	3.325	4.168	5.380	5.899	6.393	8.343
10	2.156	2.558	3.059	3.247	3.940	4.865	6.179	6.737	7.267	9.342
11	2.603	3.053	3.609	3.816	4.575	5.578	6.989	7.584	8.148	10.341
12	3.074	3.571	4.178	4.404	5.226	6.304	7.807	8.438	9.034	11.340
13	3.565	4.107	4.765	5.009	5.892	7.041	8.634	9.299	9.926	12.340
14	4.075	4.660	5.368	5.629	6.571	7.790	9.467	10.165	10.821	13.339
15	4.601	5.229	5.985	6.262	7.261	8.547	10.307	11.037	11.721	14.339
16	5.142	5.812	6.614	6.908	7.962	9.312	11.152	11.912	12.624	15.338
17	5.697	6.408	7.255	7.564	8.672	10.085	12.002	12.792	13.531	16.338
18	6.265	7.015	7.906	8.231	9.390	10.865	12.857	13.675	14.440	17.338
19	6.844	7.633	8.567	8.907	10.117	11.651	13.716	14.562	15.352	18.338
20	7.434	8.260	9.237	9.591	10.851	12.443	14.578	15.452	16.266	19.337

Table A.3 Areas under the Normal Curve



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641