COLLEGE OF ENGINEERING, PUNE

(An Autonomous Institute of Government of Maharashtra.)
SHIVAJI NAGAR, PUNE - 411 005

END Semester Examination

(MA-201) Engineering Mathematics-III

ourse: B.Tech All Branches

emester: Sem III

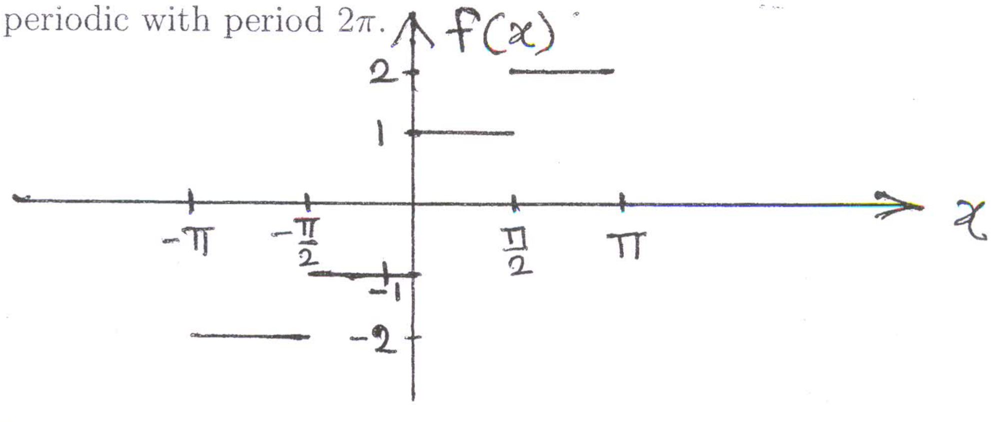
ear: 2014-2015 Max.Marks:60

ration: 3 Hours Time:- 10am-1pm **Date:24/11/2014**

Instructions: **MIS No.** | | | | | | | | | |

1. Figures to the right indicate the full marks.
2. Mobile phones and programmable calculators are strictly prohibited.
3. Writing anything on question paper is not allowed.
4. Exchange/Sharing of anything like stationery, calculator is not allowed.
5. Assume suitable data if necessary.
6. Write your MIS Number on Question Paper

Q.1 (a) Find the Fourier series of the function $f(x)$ which is assumed to be periodic with period 2π. [3]



(b) Determine the angle between the normals to the surface $xy = z^2$ at the points $(4, 1, 2)$ and $(3, 3, -3)$. [2]

(c) Evaluate $\iint_S \vec{F} \cdot \hat{n}\, dA$ where $\vec{F} = [0, x, 0]$ and $S : \vec{r}(u, v) = [u\cos v, u\sin v, u^2]$, $0 \le u \le 4$, $-\pi \le v \le \pi$. [2]

(d) Let $D_x \equiv \frac{\partial}{\partial x}$ and $D_y \equiv \frac{\partial}{\partial y}$. Suppose that the solution of the partial differential equation $(pD_x^3 + qD_x^2 D_y + rD_x D_y^2 + sD_y^3)z = 0$ is given by $z = f(y + m_1 x) + g(y + m_2 x) + h(y + m_3 x)$, where p, q, r, s are some constants; f, g, h are arbitrary functions and m_1, m_2, m_3 are the roots of the auxiliary equation $pm^3 + qm^2 + rm + s = 0$. Assuming this information, solve $(D_x^3 - 3D_x^2 D_y + 2D_x D_y^2)z = 0$. [2]

(e) Solve the following linear integral equation.

$$y(t) = \sin 2t + \int_0^t y(\tau)\ \sin 2(t - \tau)\, d\tau. \qquad [2]$$

(f) Evaluate the integral $\int_0^\infty e^{-5t}\ (t \cos t)\ dt$ using properties of Laplace transform. [2]

(g) The probability that a patient recovers from a certain disease is 0.4. Suppose 10 persons are suffering from this disease. Determine the the probability that all 10 recover from the disease. [2]

Q.2 (a) Derive the one dimensional heat equation which governs the heat flow in a long thin bar of length L with insulated lateral surface. [4]

(b) Find the steady state temperature distribution in a thin rectangular plate $0 < x < a$, $0 < y < b$ with its two faces insulated, under the boundary conditions $u(0, y) = u(a, y) = u(x, b) = 0$ and $u(x, b) = f(x) = 100$, $0 < x < a$. [4]

(c) Find the vibrations of a rectangular membrane of sides $a = 4$ ft and $b = 2$ ft, if the tension is 12.5 lb/ft, the density is 2.5 slugs/ft^2, the initial velocity is 0 and the initial displacement is $f(x, y) = 0.1(4x - x^2)(2y - y^2)$ ft. [5]

(d) Find the directional derivative of $f(x, y, z) = x^2 + y^2 - z$ at $P : (1, 1, -2)$ in the direction of $\vec{a} = [1, 1, 2]$. [2]

Q.3 (a) Suppose $\mathcal{L}\{f(t)\} = F(s)$. Assuming all necessary conditions, prove that $\mathcal{L}\{\frac{f(t)}{t}\} = \int_s^\infty F(\tilde{s})\ d\tilde{s}$. [3]

(b) Solve the following initial value problem by the Laplace transform: $y'' + y = 2t$; $y(\frac{\pi}{4}) = \frac{\pi}{2}$, $y'(\frac{\pi}{4}) = 3$. [4]

(c) Let $f(t) = \sin \omega t$ if $t > \frac{6\pi}{\omega}$ and 0 otherwise, where ω is a fixed constant. Express $f(t)$ in terms of unit step function and hence find its Laplace transform. [2]

(d) Prove that $\mathcal{L}(\delta(t - a)) = e^{-as}$. [2]

(e) Find functions $f(t)$ and $g(t)$ such that

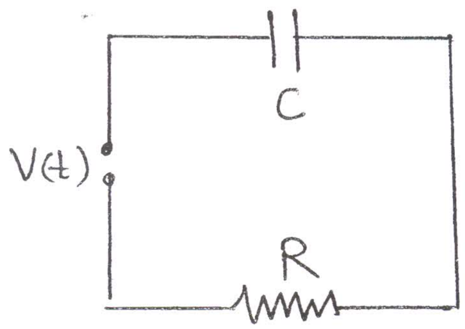
$$\mathcal{L}^{-1}\left(\frac{e^{-s}}{s}\tan^{-1}\left(\frac{s-1}{4}\right)\right) = f * g. \qquad [4]$$

OR

Find the current $i(t)$ using Laplace transform in the following $R-C$ circuit if the input voltage(electromotive force) is

$$v(t) = \begin{cases} v_0 & \text{if} \quad a < t < b; \\ 0 & \text{if} \quad Otherwise; \end{cases}$$

is applied. Current and charge are initially zero. [4]



Q.4 (a) Let $\overline{F} = -GmM\frac{\overline{r}}{r^3}$, be the Gravitational field where G, m, M are constants. Let P_1, P_2 be two points at distances s_1, s_2 from the origin. Show that the work done by the gravitational field $\overline{F}$ in moving a particle from P_1 to P_2 is
$GmM(1/s_2 - 1/s_1)$. [3]

(b) A box contains 20 screws , 5 of which are defective. If a sample of 3 screws is chosen from the box by random without replacement, find the probability that 2 screws in the sample will be defective. [2]

OR

At a certain college, the probability that a member of faculty is absent on any one day is 0.001. If there are 300 faculty members, calculate the probability that the number of faculty members absent on any one day is more than 2. [2]

(c) i. Suppose the life of a certain electronic component (in hours) is a continuous random variable with probability density function $f(x) = 20000/x^3$, if $x \geq 100$ and 0 otherwise. Find the mean life (in hours) of the electronic component. [2]

ii. Suppose the distribution of IQ scores can be approximated by a normal distribution with mean 100 and variance 225. Find the number of children with IQ greater than 120 in a school of 1800 pupils. [2]

iii. Marks scored by candidates in an examination follows normal distribution. If 44% of the candidates scored marks below 55 and 6% of the candidates scored marks above 80, find the mean and variance of marks. [2]

(d) If a sample of 30 tires of a certain kind has mean life of 35000 miles and a standard deviation of 4000 miles, can the manufacturer claim that the true mean life of such tires is greater than 30000 miles? Assume normality and use 5% level of significance. [4]

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