COLLEGE OF ENGINEERING, PUNE

(An Autonomous Institute of Government of Maharashtra.)
SHIVAJI NAGAR, PUNE - 411 005

END Semester Examination

(MA-203) Foundation of Mathematics-I

Course: B.Tech Branch: All

Semester: Sem III

Year: 2014-2015 Max.Marks:60

Duration. 3 Hours Time:- 10AM - 1PM **Date:24/11/2014**

Instructions: **MIS No.** | | | | | | | | | |

1. Figures to the right indicate the full marks.
2. Mobile phones and programmable calculators are strictly prohibited.
3. Writing anything on question paper is not allowed.
4. Exchange/Sharing of anything like stationery, calculator is not allowed.
5. Assume suitable data if necessary.
6. Write your MIS Number on Question Paper

Q1 a) State whether following statements are true or false. Justify your answer. [2M]

1. Any subset of set of linearly dependent set is linearly dependent.
2. Any superset of set of linearly dependent set is linearly dependent.

b) Check whether the following matrix A is diagonalizable.

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

If yes, then find invertible matrix P such that $A = P^{-1}DP$ where, D is a diagonal matrix. If not, give reason. [5M]

c) An elastic membrane in the x_1-x_2 plane with boundary circle $x_1^2 + x_2^2 = 1$ is stretched by an elastic deformation $\mathbf{y} = A\mathbf{x}$ where,

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \mathbf{Ax} = \begin{bmatrix} 3 & 5 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Find the principal directions and corresponding factor of extension or contraction. Explain the geometrical significance [5M]

Q2 a) For the function $f(x) = 4x^3 - x^4$, [6M]

i. determine the largest possible domain D

2. determine all critical points.
3. determine the intervals on which the function is increasing or decreasing.
4. determine local extreme values and the points at which these values occur.
5. determine the intervals on which the function is concave up / concave down by identifying the point of inflection.
6. Sketch a neat graph and hence determine the global extreme values and the points at which these values occur.

b) Sketch the possible graph of a differentiable function $y = f(x)$ through the point $(1, 1)$ if $f'(1) = 0$ and $f'(x) > 0$ for $x \neq 1$.. [1M]

c) Sketch the possible graph of a differentiable function $y = f(x)$ that has a local maxima at $(3, 3)$ and a local minima at $(1, 1)$. [1M]

d) Compute the value. Show your calculations. [1M]
$\Gamma(\frac{-1}{2})$

e) Fill in the blanks. [3M]

1. $\int_0^{\frac{\pi}{2}} \sin^p(\theta)\cos^q(\theta)\, d\theta =$ ________.

2. $\beta(m, n) = \int_0^{\infty}$ ________ dy.

3. $\int_0^{\infty} x^4 e^{-x}\, dx =$ ________.

Q3 a) Let $w = x^3 + y$ where $x = r$ and $y = r + s$. Find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$. [2M]

b) Find $\frac{\partial^2 f}{\partial x \partial y}; \frac{\partial^2 f}{\partial y \partial x}$ for $f(x, y) = xsiny + ysinx + xy$. Is $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$? [2M]

c) Find the largest and smallest values that the function $f(x, y) = 3x + 4y$ takes on the circle $x^2 + y^2 = 1$ by using Lagrange multiplier method. [4M]

d) Find all local extreme values and saddle points of $f(x, y) = 4xy + x^4 + y^4$. [4M]

Q4 a) Evaluate [2M]

$$\int_1^2 \int_y^{y^2} dxdy$$

b) Find the area of the region bounded by parabolas $x = y^2$ and $x = 2y - y^2$. [4M]

c) Change to polar co-ordinates and evaluate the integral. [5M]

$$\int_0^{\ln 2} \int_0^{\sqrt{(\ln 2)^2 - y^2}} e^{\sqrt{x^2+y^2}}\, dx\, dy$$

d) Sketch and shade the region R in x-y plane which is enclosed by the positive x-axis and spiral $r = \frac{4\theta}{3}$ $(0 \le \theta \le 2\pi)$. [1M]

Q5 a) Evaluate [2M]

$$\int_0^{2\pi}\int_0^{1}\int_r^{\sqrt{2-r^2}} r\, dz\, dr\, d\theta$$

b) Find the volume of the region in first octant bounded by the co-ordinate planes $x + z = 1$ and $y + 2z = 2$ [5M]

c) Show that centroid of the solid right circular cone is one-fourth way from base to vertex. [5M]

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