COLLEGE OF ENGINEERING, PUNE

(An Autonomous Institute of Government of Maharashtra.)
SHIVAJI NAGAR, PUNE - 411 005

END Semester Examination

(CT-201) Discrete Structures and Graph Theory

Course: B.Tech Branch: Computer Engineering/Information Technology

Semester: Sem III

Year: 2014-2015 Max.Marks:60

Duration: 3 Hours Time:- 10 to 1.00 p.m Date: 28 NOV 2014

Instructions: **MIS No.** | | | | | | | | |

1. **Figures to the right indicate the full marks.**
2. ***All the Questions are compulsory***
3. **Mobile phones and programmable calculators are strictly prohibited.**
4. **Writing anything on question paper is not allowed.**
5. **Exchange/Sharing of anything like stationery, calculator is not allowed.**
6. **Assume suitable data if necessary.**
7. ***Once the question is attempted all sub-question should be solved Consecutively***
8. ***If options are there, only first attempted will be evaluated***
9. **Write your MIS Number on Question Paper**

Q.1 A Let P and q be the propositions " Swimming at the New Jersey shore is allowed " and ' Sharks have been spotted near the shore" respectively. [3]
Express each of the compound propositions as an English sentence.

a) $P \rightarrow \neg q$
b) $\neg P \rightarrow \neg q$
c) $P \leftrightarrow \neg q$
d) $\neg P \wedge (P \neg q)$

OR

Let P and q be the propositions

P : You drive over 65 miles per hour
q : You get a speeding ticket.

Write these propositions using p and q and logical connectives

a) If you do not drive over 65 miles per hour, then you will not get a speeding ticket.
b) Driving over 65 miles per hour is sufficient for getting a speeding ticket.
c) You get a speeding ticket, but you do not drive over 65 miles per hour.
d) Whenever you get a speeding ticket, you are driving over 65 miles per hour.

B Let $L(x, y)$ be the statement "x loves y," where the domain for both x and y consists of all people in the world. Use quantifiers to express each of these [3] statements.

a) Everybody loves Jerry.
b) Everybody loves somebody.
c) There is somebody whom everybody loves.
d) Nobody loves everybody.
e) There is somebody whom Lydia does not love.
f) There is somebody whom no one loves.
g)There is exactly one person whom everybody loves.
h) Everyone loves himself or herself

C Use rules of inference to show that if [4]

$\forall x(P(x) \rightarrow (Q(x) \wedge S(x)))$ and $\forall x(P(x) \wedge R(x))$ are true, then

$\forall x(R(x) \wedge S(x))$ is true

Q.2 A [4]

Prove that $1^2 + 3^2 + 5^2 + \cdots + (2n + 1)^2 = (n + 1)(2n + 1)(2n + 3)/3$
whenever n is a nonnegative integer.

B Draw the Venn diagrams for each of these combinations of the sets A, B, and C. [3]
a) $A \cap (B - C)$
b) $(A \cap B) \cup (A \cap C)$
c) $(A \cap \bar{B}) \cup (A \cap \bar{C})$

C Justify whether following are function or not. [3]
a) $f(x) = 1/x$
b) $f(x) = \sqrt{x}$
c) $f(x) = \pm\sqrt{x^2 + 1}$

Q.3 A Show that set Z of all integers is countable [2]

B Consider the functions **R->R** where [2]
$f(x) = 3x+7$ for all $x \in R$.
$g(x) = x^4 - x$
Determine whether these functions are one-to-one or not?

C Consider the set **Z** of integers. Define aRb by $b = a^r$ for some positive integer r. [3]
Show that R is a partial order on **Z**.

D If R be a relation on the set of integers Z defined by [3]
R = {(x,y) : $x \in Z$, $y \in Z$, $(x-y)$ *is divisible by 6*} .
Then prove that R is an equivalence relation

Q.4 A The relation R= {(a,a),(a,b), (b,a), (b,b), (c,c)} on A {a,b,c} is an equivalence relation. [2]
Find the equivalence class of each element in A.

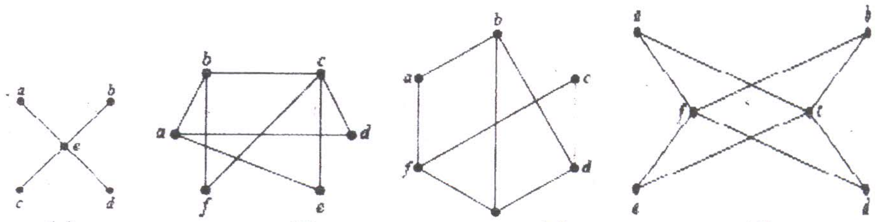
B Let $A = \{a, b, c\}, B = \{x, y, z\}, C = \{r, s, t\}$. Let $f: A \rightarrow B$ and $g: B \rightarrow C$ be defined by: [3]
$f = \{(a, y)(b, x), (c, y)\}$ and $g = \{(x, s), (y, t), (z, r)\}$.

Find: (*a*) composition function $g \circ f: A \rightarrow C$; (*b*) Im($f$), Im($g$), Im($g \circ f$).(Image of f, G and composition function)

C Answer these questions for the poset ({{1}, {2}, {4}, {1, 2}, {1, 4}, {2, 4}, {3, 4}, {1, 3, 4}, {2, 3, 4}}, ⊆). [3]
Draw the Hase diagram
a) Find the maximal elements.
b) Find the minimal elements.
c) Is there a greatest element?
d) Is there a least element?

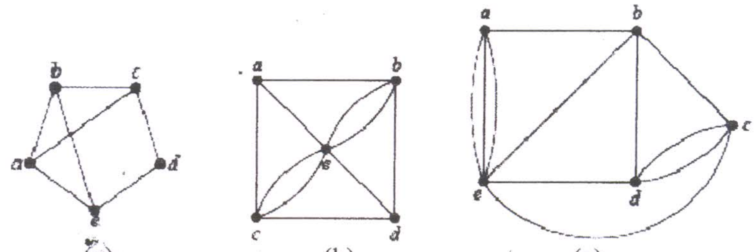
e) Find all upper bounds of {{2}, {4}}.
f) Find the least upper bound of {{2}, {4}}, if it exists.
g) Find all lower bounds of {{1, 3, 4}, {2, 3, 4}}.
h) Find the greatest lower bound of {{1, 3, 4}, {2, 3, 4}}, if it exists.

D Which of these this is bi-partie graph [2]

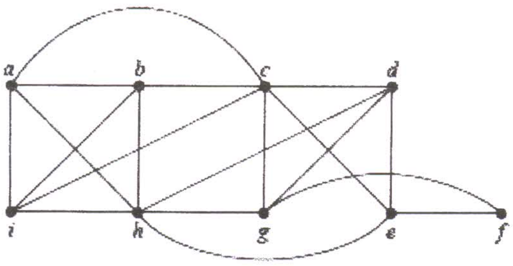


(a) (b) (c) (d)

Q.5 A [3]



(a) (b) (c)

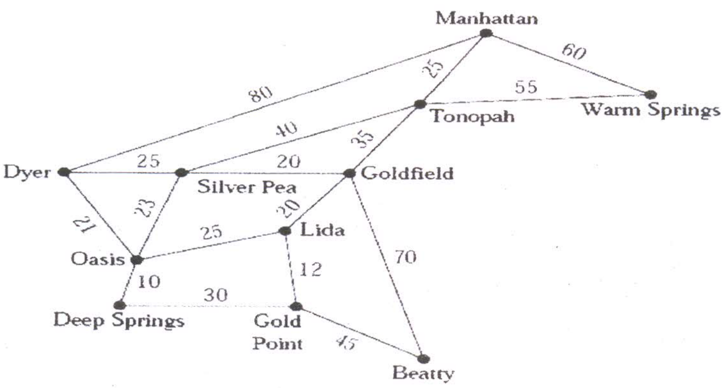


(d)

Find out which of the above graph has Euler Circuit or path. Write down that path/circuit if it exists

B Schedule the final exams for Math 115, Math 116, Math 185, Math 195, CS 101, CS 102, CS 273, and CS 473, using the fewest number of different time slots,if there are no students taking both Math 115 and CS 473, both Math 116 and CS 473, both Math 195 and CS 101, both Math 195 and CS 102, both Math 115 and Math 116, both Math 115 and Math 185, and both Math 185 and Math 195, but there are students in every other pair of courses. [4]

C The roads represented by this graph are all unpaved. The lengths of the roads between pairs of towns are represented by edge weights. Which roads should be paved so that there is a path of paved roads between each pair of towns so that a minimum road length is paved? [3]



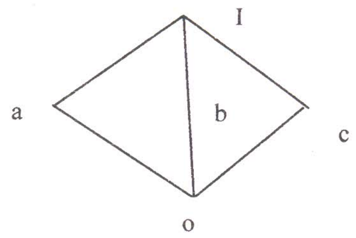
Q.6 **A** [5]

Consider the set Z together with the binary operation of $\oplus$ and $\odot$, which are defined by

$X \oplus Y = X+Y-1$, $x \odot y = X+Y-XY$

Prove that $(Z, \oplus, \odot)$ is a ring.

B Show that the lattice shown in figure is not a distributive lattice [3]



C [2]

Let A={a,b} and * is a binary operation on it. Which of the following tables define a semigroup on A? Which define a monoid on A?

*	a	b
a	a	b
b	a	a

*	a	b
a	a	b
b	b	b